Assignment-set 3 Introduction to Dynamical Systems 2016

Deadline to hand in: 28 November 2016, 11:15u

1.) Let $\phi(t; a)$ define a smooth n-dimensional flow in $\mathbf{R}^n$ and let A be a subset of $\mathbf{R}^n$. The omega limit set of the set A is defined as the union of all $\omega(a)$ over all $a \in A$, i.e. $\omega(A) = \cup_{a \in A} \omega(a)$. Since for a given a_0, $\omega(a_0)$ also is a subset of $\mathbf{R}^n$, one can thus define $\omega(\omega(a_0))$. Is it true that $\omega(\omega(a_0)) = \omega(a_0)$? If you agree, give a proof; if not, give a counter example.

2.) Exercise 19, page 70 of the book.

3.) Consider the system

$$\begin{cases} \dot{x} &= -xy, \\ \dot{y} &= y(2x-1)f(x,y), \end{cases} \tag{1}$$

with$f : \mathbf{R}^2 \to \mathbf{R}$ Lipschitz continuous and $1 \le f(x,y) \le 2$.

(a) Determine the sets Γ_x and Γ_y at which $\dot{x} = 0$ and $\dot{y} = 0$, respectively; determine all critical points of (1).

(b) Let $\gamma^+(t)$ $(t \ge 0)$ be the solution of (1) with $\gamma^+(0) = (1,1)$. Show that $\omega(\gamma^+) = \omega((1,1)) = (x^*, 0)$ for some $x^* \in (0, \frac{1}{2})$.
Hint: Show that $\gamma^+(t)$ cannot cross through line segment that connects $(1,1)$ to $(\frac{1}{2}, 2)$ and use (a).

(c) Take $f(x,y) \equiv 1$. Give an explicit expression for the orbit γ^+ and derive a relation that determines x^* uniquely.

(d) Let $I_f \subset (0, \frac{1}{2})$ be the set of all x^*'s that may appear in the limit set $\omega((1,1))$ (for functions $f(x,y)$ that satisfy the conditions in (1)). Determine a set $S \subset (0, \frac{1}{2})$ that contains I_f and try to make S as small as possible. Can you obtain the preferred situation $S = I_f$? If so, prove that this is indeed the case.

4.) Exercise 16, page 163 of the book.