Assignment-set 2 Introduction to Dynamical Systems 2016

Deadline to hand in: 31 October 2016, 11.15u

1.) Consider for $a, b \in \mathbf{R}$ and $n \in \mathbf{N}$ the two-dimensional system,

$$\left\{ \begin{array}{rcrcrcr} \dot{x} & = & -ax & + & by & + & ay^{n+1}, \\ \dot{y} & = & (b-a)x & & & + & bxy^n. \end{array} \right. \tag{1}$$

(a) Take $n = 1$. Determine for all $a, b \in \mathbf{R}$ the stability of the critical point $(0, 0)$. Give a sketch of the outcome in the (a, b)-parameter plane.
Hint: First determine the eigenvalues $\lambda_{1,2} = \lambda_{1,2}(a, b)$ of the linearization of (1) about $(0, 0)$. Next, consider the special cases – i.e. the subsets in the (a, b)-plane for which $\text{Re}(\lambda_j(a, b)) = 0$ (for $j = 1$ or $j = 2$) – separately; use the definition of (in)stability for these special cases.

(b) Take $n = 2$ and again determine the stability of $(0, 0)$ for all $a, b \in \mathbf{R}$.

2.) (a) Show that $\bar{x} = 0$ is an asymptotically stable solution of $\dot{x} = -x^3$ ($\in \mathbf{R}$). Use the definition! (see page 118 of the book).

(b) Is this also true for the *complex* equation $\dot{z} = -z^3$ (with $z \in \mathbf{C}$)? Use again the definition.
Hint: Write this complex equation as a 2-dimensional real system by introducing the real and imaginary parts of z, i.e. by setting $z = x + iy$ and deriving equations for $\dot{x}$ and $\dot{y}$. Make a sketch of the phase portrait. Note that *in*stability is defined as *not* being stable (page 116 book).

3.) (a) Exercise 9 on pages 161, 162 of the book – see also the example on page 124.

(b) Exercise 10 on page 162 of the book.
Note: This provides an alternative proof of Theorem 4.6.