Opfriscursus – antwoorden goniometrie

$\sin(\frac{1}{3}\pi) = \frac{1}{2}\sqrt{3}$, $\sin(1\frac{1}{3}\pi) = -\frac{1}{2}\sqrt{3}$, $\cos(3\pi) = -1$

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a) $\sin(2x) = \sin(3x)$ $\quad 2x = 3x + k2\pi$ of $2x = \pi - 3x + k2\pi$

$x = -k2\pi$ of $x = \frac{1}{5}\pi + k\frac{2}{5}\pi$, $k \in \mathbb{Z}$.

b) $\sin^2 x = \frac{1}{2}$ $\quad \sin x = \frac{1}{2}\sqrt{2}$ of $\sin x = -\frac{1}{2}\sqrt{2}$

$x = \frac{\pi}{4} + k2\pi$, $x = \frac{3\pi}{4} + k2\pi$, $\frac{5\pi}{4} + k2\pi$ of $\frac{7\pi}{4} + k2\pi$

ofwel $x = \frac{\pi}{4} + k\frac{\pi}{2}$, $k \in \mathbb{Z}$.

c) $\cos^2 x + \sin x = \frac{1}{4}$ $\quad 1 - \sin^2 x + \sin x = \frac{1}{4}$

$(\sin x)^2 - \sin x - \frac{3}{4} = 0$

$(\sin x - \frac{3}{2})(\sin x + \frac{1}{2}) = 0$

$\sin x = \frac{3}{2}$ of $\sin x = -\frac{1}{2}$

$\times \qquad x = \frac{7}{6}\pi + k2\pi$ of $\frac{11}{6}\pi + k2\pi$, $k \in \mathbb{Z}$.

d) $\sin(2x) = 2\cos x$ $\quad 2\sin x \cos x = 2\cos x$

$2\cos x(\sin x - 1) = 0$

$\cos x = 0$ of $\sin x = 1$

$x = \frac{\pi}{2} + k\pi$ of $x = \frac{\pi}{2} + k2\pi$

ofwel $x = \frac{\pi}{2} + k\pi$, $k \in \mathbb{Z}$.

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a) $y = 2\sin x$ grafiek als van $\sin x$, maar amplitude $\times 2$

$y = 2 + \sin x$ —— " —— 2 omhoog verschoven

$y = \sin(x-2)$ —— " —— 2 naar rechts verschoven

$y = \sin(2x)$ —— " —— 2 keer zo snel, ofwel periode π i.p.v. 2π

b) $y = a + b\sin(c(x-d))$ met

$a = \frac{max + min}{2} = \frac{5+1}{2} = 3$, $b = \frac{max - min}{2} = \frac{5-1}{2} = 2$,

$c = \frac{2\pi}{periode} = \frac{2\pi}{14-4} = \frac{\pi}{5}$, d = verschuiving naar rechts dus

$d = 4 -$ kwart periode $= 4 - \frac{10}{4} = 1\frac{1}{2}$. Dus $y = 3 + 2\sin(\frac{\pi}{5}(x - 1\frac{1}{2}))$.

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a) $\sin x = 0 \qquad x = k\pi$

$\sin x = 1 \qquad x = \frac{\pi}{2} + k2\pi$

$\sin x = -1 \qquad x = \frac{3\pi}{2} + k2\pi$

$\sin x = \frac{1}{2} \qquad x = \frac{\pi}{6} + k2\pi$ of $x = \frac{5}{6}\pi + k2\pi$ $\qquad$, $k \in \mathbb{Z}$.

$\sin x = \frac{1}{2}\sqrt{2} \qquad x = \frac{\pi}{4} + k2\pi$ of $x = \frac{3\pi}{4} + k2\pi$

$\sin x = \frac{1}{2}\sqrt{3} \qquad x = \frac{\pi}{3} + k2\pi$ of $x = \frac{2\pi}{3} + k2\pi$

b) $1 - 2\cos(3x) = 0$ $\quad \cos(3x) = \frac{1}{2}$, $3x = \frac{\pi}{3} + k2\pi$ of $3x = -\frac{\pi}{3} + k2\pi$

$x = \frac{\pi}{9} + k\frac{2}{3}\pi$ of $x = -\frac{\pi}{9} + k\frac{2}{3}\pi$, $k \in \mathbb{Z}$.

c) $\sin(\frac{1}{2}\pi - 3x) = \cos(2x)$ $\quad \cos(3x) = \cos(2x)$

$3x = 2x + k2\pi$ of $3x = -2x + k2\pi$

$x = k2\pi$ of $x = k\frac{2}{5}\pi$, $k \in \mathbb{Z}$.

d) $1 + (\sin\frac{1}{6}\pi)^2 + \cos(\frac{1}{6}\pi n) = \frac{1}{4}$ $\quad 1 + (\frac{1}{2})^2 + \cos(\frac{1}{6}\pi n) = \frac{1}{4}$

$\cos(\frac{1}{6}\pi n) = -1$

$\frac{1}{6}\pi n = \pi + 2k\pi$, $n = 6 + 12k, k \in \mathbb{Z}$.

$f(x) = 1 - \cos^2 x \qquad f'(x) = -2\cos x \cdot -\sin x = \sin(2x)$

$g(x) = 2 + 3\cos(2x) \qquad g'(x) = -3\sin(2x) \cdot 2 = -6\sin(2x)$

$h(x) = (\sin(2x))^2 \qquad h'(x) = 2\sin(2x) \cdot \cos(2x) \cdot 2 = 4\sin(2x)\cos(2x)$

$j(x) = \frac{\sin x}{1 + \sin x} \qquad j'(x) = \frac{\cos x(1 + \sin x) - \sin x \cos x}{(1 + \sin x)^2} = \frac{\cos x}{(1 + \sin x)^2}$

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a) max $y = 2\cos(4x) - 1$ $\quad$ max als $\cos(4x) = 1$ ofwel $4x = k2\pi$

$x = k\frac{\pi}{2}, k \in \mathbb{Z}$;

en dan $y = 1$.

b) max $y = 2 + 3\sin(\frac{1}{6}\pi(x+5))$ $\quad$ max als $\sin(\frac{1}{6}\pi(x+5)) = 1$

ofwel $\frac{1}{6}\pi(x+5) = \frac{\pi}{2} + k2\pi$

$x = -5 + 3 + 12k$

$x = -2 + 12k$, $k \in \mathbb{Z}$

en dan $y = 5$.

c) max $y = 4\sin^2(x) - 4\sin x$ $\quad$ max $4u^2 - 4u$ op $[-1, 1]$:

$\frac{d}{du}(4u^2 - 4u) = 8u - 4 = 0 \rightarrow u = \frac{1}{2}$

dalparabool, dus max voor $u = -1$ ofwel $\sin x = -1$

ofwel $x = \frac{3\pi}{2} + k2\pi$ en dan $y = 8$.

a) $\int_0^{\frac{1}{3}\pi} \sin(3x)\,dx = -\frac{1}{3}\cos(3x)\Big|_0^{\frac{1}{3}\pi} = -\frac{1}{3}(-1-1) = \frac{2}{3}$

b) Opp tussen x-as en $y = \sin\left(\frac{1}{6}\pi x\right)$ tussen $x=0$ en eerstvolgende snijpunt: $\sin\left(\frac{1}{6}\pi x\right)=0$: $\frac{1}{6}\pi x = k\pi$, $x = 6k$,

dus $\int_0^6 \sin\left(\frac{1}{6}\pi x\right)dx = -\frac{6}{\pi}\cos\left(\frac{1}{6}\pi x\right)\Big|_0^6 = -\frac{6}{\pi}(-1-1) = \frac{12}{\pi}$.

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a) $y = 1 + (\sin x)^2 = 1 + \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right)$ $\qquad \cos 2x = 1 - 2\sin^2 x$

$= 1\frac{1}{2} - \frac{1}{2}\cos(2x)$.

b) $y = (\sin 2x)^2 = (2\sin x\cos x)^2 = 2\sin^2 x(1-\sin^2 x) =$

$= 2\sin^2 x - 2\sin^4 x$.

c) $y = \cos 4x = 2(\cos 2x)^2 - 1$ $\qquad \cos 2x = 2\cos^2 x - 1$

$= 2(\cos^2 x - 1)^2 - 1 = 2(4\cos^4 x - 4\cos^2 x + 1) - 1$

$= 8\cos^4 x - 8\cos^2 x + 1$.

d) $y = 1 + \sin^2 x + \cos(2x) = 1 + \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) + \cos 2x$

$= 1\frac{1}{2} + \frac{1}{2}\cos 2x = 1\frac{1}{2} + \frac{1}{2}\sin\left(\frac{\pi}{2} - 2x\right) = 1\frac{1}{2} - \frac{1}{2}\sin\left(2x - \frac{\pi}{2}\right)$

$= 1\frac{1}{2} - \frac{1}{2}\sin\left(2\left(x - \frac{\pi}{4}\right)\right)$.

e) $f(x) = \cos x\cos\left(\frac{1}{3}\pi - x\right)$

$f'(x) = -\sin x\cos\left(\frac{1}{3}\pi - x\right) + \cos x\sin\left(\frac{1}{3}\pi - x\right)$

$= \sin\left(\left(\frac{1}{3}\pi - x\right) - x\right) = \sin\left(\frac{1}{3}\pi - 2x\right)$

f) $\left(3\sin 2\pi t - 2\sin\frac{1}{6}\pi t\right)^2 + \left(3\cos 2\pi t - 2\cos\frac{1}{6}\pi t\right)^2$

$= 9\sin^2 2\pi t - 12\sin 2\pi t\sin\frac{1}{6}\pi t + 4\sin^2\frac{1}{6}\pi t$

$+ 9\cos^2 2\pi t - 12\cos 2\pi t\cos\frac{1}{6}\pi t + 4\cos^2\frac{1}{6}\pi t$

$= 9 + 4 - 12\cos\left(2\pi t - \frac{1}{6}\pi t\right) = 13 - 12\cos\left(\frac{11}{6}\pi t\right)$.

g) $y = \sin x + \sin(3x) = \sin x + \sin(x + 2x)$

$= \sin x + \sin x\cos 2x + \cos x\sin 2x$

$= \sin x + \sin x(1 - 2\sin^2 x) + \cos x \cdot 2\sin x\cos x$

$= \sin x + \sin x - 2\sin^3 x + 2\sin x(1 - \sin^2 x)$

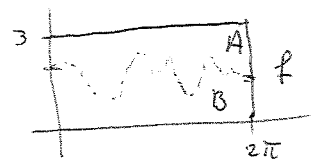
$= 4\sin x - 4\sin^3 x$.

Bijlage 2: $f_n(x) = 1 + \sin^2 x + \cos nx$ op $[0, 2\pi]$, $n \in \mathbb{N}$.

- Als de grafiek van f_n gaat door $\left(\frac{1}{6}\pi, \frac{1}{4}\right)$, dan $1 + \left(\sin\frac{1}{6}\pi\right)^2 + \cos\left(\frac{n}{6}\pi\right) = \frac{1}{4}$, dus (zie eerder) $n = 6 + 12k$, dus $n = 6, 18, 30, 42, \ldots$.
- $f_4(x) = 1 + \sin^2 x + \cos 4x = 1 + \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) + \cos 4x$

  $= 1\frac{1}{2} - \frac{1}{2}\cos x + \cos 4x$.
- $f_4(x) = 1 + \sin^2 x + \cos 4x \leq 1 + 1 + 1 = 3$ en

  $f_4(x) \geq 1 + 0 - 1 = 0$.



Te bewijzen: Opp A = Opp B ofwel

$\int_0^{2\pi}(3 - f_4(x))\,dx = \int_0^{2\pi} f_4(x)\,dx$.

Er geldt:

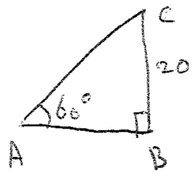
$\int_0^{2\pi}(3 - f_4(x))\,dx = \int_0^{2\pi}\left(1\frac{1}{2} + \frac{1}{2}\cos x - \cos 4x\right)dx$

$= \int_0^{2\pi} 1\frac{1}{2}\,dx + \frac{1}{2}\underbrace{\int_0^{2\pi}\cos x\,dx}_{=0} - \underbrace{\int_0^{2\pi}\cos 4x\,dx}_{=0}$

$= \int_0^{2\pi} 1\frac{1}{2}\,dx - \frac{1}{2}\int_0^{2\pi}\cos x\,dx + \int_0^{2\pi}\cos 4x\,dx$

$= \int_0^{2\pi}\left(1\frac{1}{2} - \frac{1}{2}\cos x + \cos 4x\right)dx = \int_0^{2\pi} f_4(x)\,dx$.

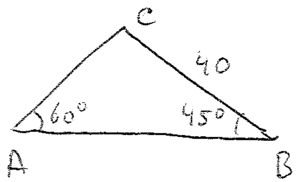
(rechthoekige driehoek)



AC? $\frac{20}{AC} = \sin 60° = \frac{1}{2}\sqrt{3}$

dus $AC = \frac{20}{\frac{1}{2}\sqrt{3}} = \frac{40}{3}\sqrt{3}$.

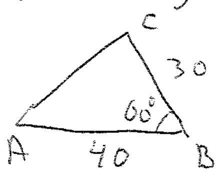
(sinusregel)



AC? $\frac{AC}{\sin 45°} = \frac{40}{\sin 60°}$, dus $AC = \frac{40}{\frac{1}{2}\sqrt{3}} \frac{1}{2}\sqrt{2}$

$= \frac{20}{3}\sqrt{6}$.

(cosinusregel)



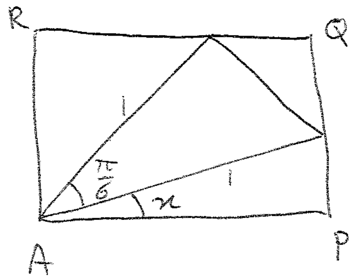
AC? $AC^2 = 40^2 + 30^2 - 2 \times 40 \times 30 \times \cos 60°$

$= 1600 + 900 - 2 \times 1200 \times \frac{1}{2}$

$= 1300$

dus $AC = \sqrt{1300} = 10\sqrt{13}$.

Bijlage 1:

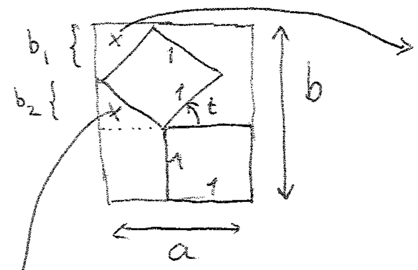


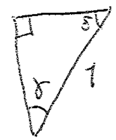
Opp APQR ?

$AP = \cos x$, $\angle RAC = \frac{\pi}{2} - \frac{1}{6}\pi - x = \frac{1}{3}\pi - x$,

dus $AR = \cos(\frac{1}{3}\pi - x)$.

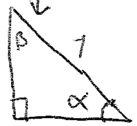
Dus $O(x) = AP \times AR = \cos x \cdot \cos(\frac{1}{3}\pi - x)$.





$\gamma = \pi - \frac{\pi}{2} - \beta = \frac{\pi}{2} - t$

dus $\delta = t$



$\alpha = \pi - \frac{\pi}{2} - t = \frac{\pi}{2} - t$

dus $\beta = t$.

Dus $a = 1 + \sin t$

Dus $b_1 = \sin t$ en $b_2 = \cos t$, dus

$b = 1 + \sin t + \cos t$.

Dus $R = a \times b = (1 + \sin t)(1 + \sin t + \cos t)$.
