Homework 20/11

1 (for half the mark for this week)

Hand in 2 new problems from 5.4 – 5.34 and 7.5 – 7.17, and

2 (for the other half of the mark for this week)

Pick $K = \mathbf{Q}(\sqrt{-\text{mynumber}})$ imaginary quadratic. Number is: birthday in any format, e.g., 12 March can be 312 or 1203, or if the numbers are too easy, you may (with explanation) substitute e.g. your mother's birthday. Let $D = \text{disc}(K) < 0$, and

$$\mathcal{O}_K = \begin{cases} \mathbf{Z}[\sqrt{D/4}] & \text{if } 4 \mid D \\ \mathbf{Z}[\frac{1+\sqrt{D}}{2}] & \text{if } D \equiv 1(4) \end{cases}$$

a) Prove $\mathcal{O}_K^* = \{\pm 1\}$ (if not, then your example definitely is too easy! why?)

b) Compute Cl_K

Don't take 1203, as that was treated in class. Note that your mark depends not only on correctness, but also on whether your number was sufficiently challenging.

Homework 27/11

3

Pick *your* $f \in \mathbf{Z}[X]$ (monic irreducible) as *challenging* as you can. Let $K = \mathbf{Q}(\alpha)$, where α is a root of f. Don't hand in the same field as anyone else! Compute

- Cl_K (i.e., give generators and the structure of this finite abelian group)
- $\mathcal{O}_K^*$ (i.e., give a system of fundamental units)

Check your answer using the zeta function. You don't need to prove error terms for your estimate for the Euler product, it suffices to estimate it as it is done in the lecture and the lecture notes, for which you may use `http://www.few.vu.nl/~streng/ant/eulerproduct.txt`.

Note that your mark depends not only on correctness, but also on how challenging your polynomial was! For example, it should *not* be quadratic, and higher unit ranks get higher marks.