

# Positivity and Noncommutative Analysis

26 – 28 September, 2018  
TU Delft



**Local organising committee:**

Marcel de Jeu  
Jan van Neerven

**Financial support from:**



# General information

## Programme

|               | Wednesday       | Thursday        | Friday         |
|---------------|-----------------|-----------------|----------------|
| 9:00 - 10:00  | Arrival         |                 |                |
| 10:00 - 10:10 | Welcome         |                 |                |
| 10:10 - 10:50 | Wolfgang Arendt | Fedor Sukochev  | Ton Schep      |
| 10:50 - 11:20 | Coffee          |                 |                |
| 11:20 - 12:00 | Werner Ricker   | Martijn Caspers | Jan Rozendaal  |
| 12:30 - 13:30 | Lunch           |                 |                |
| 14:00 - 14:40 | Tony Wickstead  | Markus Haase    | Onno van Gaans |
| 14:40 - 15:10 | Coffee          |                 |                |
| 15:10 - 15:50 | Koos Grobler    | Erik Koelink    | Mark Veraar    |
| 15:50 - 16:20 | Coffee          |                 |                |
| 16:20 - 17:00 | Jurie Conradie  | Guido Sweers    | t.b.a.         |
| 17:00 -       |                 |                 | Reception      |
| 19:00 -       |                 | Dinner          |                |

## Location

All talks will take place in the Snijderszaal at the EEMCS (EWI) building on the TU Delft campus. The address is **Mekelweg 4, 2628 CD, Delft**. By the university's numbering scheme it is **Building 36**. It is the tallest building on campus and has a striking red and blue design. On foot it takes a 30 minute walk from the railway station and from the centre of Delft. The building can also be reached by bus 55 from the railway station (get out at **TU - Mekelpark** or **TU - Aula**). Live bus times and public transport information are available at [9292.nl](http://9292.nl).

## Lunch, conference dinner, reception

Lunch will be provided every day at 12:30 on the first floor of the EEMCS building. The conference dinner for the invited speakers, participants from abroad, invited guests, and Ben's TUD colleagues is on Thursday at 19:00 at **Restaurant van der Dussen** in the centre of Delft (Bagijnhof 118). A reception will be held on Friday at 17:00 on the first floor of the EEMCS building.

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## Disjointness preserving operators and isospectral Laplacians

Wolfgang Arendt

Disjointness preserving operators have kept the attention of the positivity community for quite a while. A remarkable result by Ben de Pagter, now 34 years old, shows that a bounded disjointness preserving operator on a Banach lattice is automatically order bounded. More recently, these operators were encountered in a completely different context. A famous example by Gordon, Webb and Wolpert to Kac's question "Can you hear the shape of a drum" consists of two isospectral polygons which are not congruent. The polygon is quite simple to describe, 7 triangles are put together in two different ways. The original proof showing that the eigenvalues are the same involves a lot of differential geometry. Berard and others gave a down-to-earth proof which we analyzed again in a joint work with James Kennedy and Tom ter Elst. It turns out that isospectrality can be best proved by constructing a bijective operator intertwining the two Laplacians. This intertwining operator translates, turns and superposes triangles. And in fact, one of the striking results is that, if there is no superposition, i.e., if a disjointness preserving operator intertwines, then it is a composition operator, and in fact composition by a congruence. This result is the content of joint work with James Kennedy on which we will report.

References:

- [1] Arendt, W.; ter Elst, A.F.M.; Kennedy, J.B.: Analytical aspects of isospectral drums. *Oper. Matrices* 8 (2014), no. 1, 255–277
- [2] Arendt, W.; Kennedy, J.B.: Disjointness preserving operators and isospectral Laplacians. Preprint.
- [3] de Pagter, B.: A note on disjointness preserving operators. *Proc. Amer. Math. Soc.* 90 (1984), no. 4, 543–549.

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## Gradient forms and strong solidity of free quantum groups

Martijn Caspers

In their fundamental paper Ozawa and Popa introduced the notion of strongly solid von Neumann algebras: the von Neumann algebra generated by the normalizer of any amenable von Neumann subalgebra is amenable again. They proved that the free group factors are strongly solid, in particular implying that they do not have a Cartan subalgebra, that they are prime factors and that they cannot be written as crossed products. In this talk we show that semi-group theory can be used to obtain strong solidity results. In particular we show that arbitrary free quantum groups are strongly solid. A key result in the proof is the study of the "gradient bimodule" associated with a natural semi-group on the free quantum groups and to show that it is weakly contained in the coarse bimodule (or "biregular bimodule").

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## **A medley of Lebesgue topologies on normed Riesz spaces**

Jurie Conradie

A net  $(x_\alpha)$  in a Riesz space  $E$  is unboundedly order convergent to 0 if for every  $u$  in  $E^+$ ,  $|x_\alpha| \wedge u \rightarrow 0$  in order (written  $x_\alpha \xrightarrow{uo} 0$ ). A locally solid topology  $\tau$  on  $E$  is a Lebesgue topology if for every order-bounded net  $(x_\alpha)$  in  $E$ ,  $x_\alpha \xrightarrow{uo} 0$  implies that  $x_\alpha \xrightarrow{\tau} 0$ . In a normed Riesz space  $E$ , we call a locally solid topology  $\tau$  on  $E$  uniformly Lebesgue if for every norm-bounded net  $(x_\alpha)$ ,  $x_\alpha \xrightarrow{uo} 0$  implies that  $x_\alpha \xrightarrow{\tau} 0$ . We identify the finest Lebesgue and uniformly Lebesgue topologies on  $E$ , and show that they may be regarded as inductive limit topologies, and that the coarsest Hausdorff Lebesgue topology on  $E$  enters into the picture.

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## **Disjointness preserving operators on pre-Riesz spaces and a theorem of Huijsmans and de Pagter**

Onno van Gaans

One of the amazing theorems concerning operators on Banach lattices is the theorem that that the inverse of a bijective disjointness preserving operator is disjointness preserving. A refined version of this statement, which clarifies the role of the underlying spaces, is due to Huijsmans and de Pagter (1993).

Several years later, Kitover raised the question whether this result can be generalized to a setting with more general partially ordered vector spaces than vector lattices. It turns out that pre-Riesz spaces provide an appropriate setting. We will briefly review pre-Riesz spaces and the corresponding embedding theory and show how it can be used to generalize the theorem by Huijsmans and de Pagter.

The talk is based on joint work with Anke Kalauch (TU Dresden) and Feng Zhang (Leiden University).

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## **Stochastic processes in Riesz spaces: Girsanov's theorem**

Koos Grobler

Girsanov's theorem arises from the problem of pricing options. We discuss this problem, show why the Girsanov theorem is needed and where risk-free measures enter the scene. We then introduce stochastic processes in Riesz spaces and show how the Girsanov theorem can be formulated and proved in this measure-free environment.

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## **Asymptotics of positive semigroups via the semigroup at infinity**

Markus Haase

Theorems assuring the strong convergence of a positive semigroup on a Banach lattice have a long tradition. One of the most intriguing instances is Greiner's theorem about semigroups on  $L^p$  containing a kernel operator. Built on earlier works of Keicher, Arendt, and Gerlach, far-reaching generalizations of this and many other results have been recently obtained by Gerlach and Glück.

In my talk I will explain the main ingredients of these recent findings and push them towards

a further simplification and generalization. (Joint work with Jochen Glück.)

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## Special functions related to group theory: multivariable matrix-valued orthogonal polynomials

Erik Koelink

The interplay between special functions and representations of groups is a very successful example of interaction between two fields of mathematics. This interplay has led to many applications in other fields in mathematics and physics as well. In the 1940s M.G. Krein introduced single-variable matrix-valued orthogonal polynomials in his study of spectral analysis of certain operators and his study of the related moment problem. Only much later the relation between explicit families of single-variable matrix-valued orthogonal polynomials and representation theory has been established. The goal of the lecture is to indicate how to extend this relation in order to obtain an explicit family of multivariable matrix-valued orthogonal polynomials. We do so by studying matrix-valued spherical functions for the group  $SU(n+1)$ , which we view as a symmetric space. These functions are matrix-valued analogues of the characters, which are single-variable Chebyshev polynomials for  $n=1$  and Koornwinder's 2-variable orthogonal polynomials on Steiner's hypocycloid for  $n=2$ , and more generally special cases of Heckman-Opdam polynomials for arbitrary  $n$ . (Joint work with Maarten van Pruijssen (U Paderborn, Germany) and Pablo Román (UN Córdoba, Argentina)).

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## Convolution operators in discrete Cesàro spaces

Werner Ricker

The discrete Cesàro spaces  $\text{ces}(p)$ ,  $1 < p < \infty$ , arise from the classical sequence spaces  $\ell^p$ ,  $1 < p < \infty$ , via the process of averaging. A sequence  $b \in \mathbb{C}^{\mathbb{N}}$  is called a  $p$ -multiplier for  $\text{ces}(p)$  if the convolution  $a * b \in \text{ces}(p)$  for every  $a \in \text{ces}(p)$ . This talk will discuss various properties of such convolution operators acting in  $\text{ces}(p)$ . The situation is *very different* than for convolution operators in the spaces  $\ell^p$ .

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## Optimal decay rates for semigroups

Jan Rozendaal

Over the past decade there has been a lot of interest in polynomial decay properties of solutions to various PDEs, most notably damped wave equations. Such polynomial decay rates can be studied effectively using the framework of semigroup theory, by exploiting links between polynomial decay properties of a  $C_0$ -semigroup and delicate spectral properties of the generator of the semigroup.

In this talk I will present several results about the optimal connection between semigroup decay and spectral properties of the generator. I will also discuss how this connection is related to vector-valued harmonic analysis, through the recently developed theory of operator-valued  $(L^p, L^q)$  Fourier multipliers.

This is joint work with Mark Veraar (Delft University of Technology), and with David

Seifert (University of Oxford) and Reinhard Stahn (TU Dresden).

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## **Regular states and the regular algebra numerical range**

Anton Schep

Let  $E$  be a complex Dedekind complete Banach lattice and let  $\mathcal{L}_r(E)$  denote the Banach lattice algebra of regular operators on  $E$ . Then a bounded linear functional  $\Phi : \mathcal{L}_r(E) \rightarrow \mathcal{C}$  is called a regular state if  $\Phi(I) = \|\Phi\| = 1$ . Basic properties of regular states are derived and a detailed description of regular states is obtained for finite dimensional  $\ell_p(n)$ -spaces. The regular states define the regular numerical algebra range. We describe the regular algebra numerical range for operators  $T$  in the center  $Z(E)$  of  $E$  and for operators  $T$  disjoint with the center. Using the description of regular states on  $\ell_p(n)$ , we characterize the regular numerical range preserving linear operators on  $\mathcal{L}_r(\ell_p(n))$ .

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## **The residue formula and Connes' trace theorem**

Fedor Sukochev

In 1989, Alain Connes showed in his famous trace theorem that the notion of integration on a compact manifold  $M$  is preserved in noncommutative geometry using a class of singular traces on pseudo-differential operators of order  $-d$  on  $L_2(M)$ . However, when  $M$  is only locally compact, technicalities ensue. In this talk, we discuss how the use of residue theorems can overcome these technicalities. Joint work with D. Potapov, D. Vella and D. Zanin.

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## **Fun with fourth order eigenfunctions**

Guido Sweers

Usually the first eigenvalue problem that one encounters in PDE is  $-\Delta\varphi = \lambda\varphi$  inside a domain  $\Omega \subset \mathbb{R}^n$  with  $\varphi = 0$  on its boundary. For all decent domains there are countably many eigenvalues  $\{\lambda_i\}_{i \in \mathbb{N}}$ , which lie in  $\mathbb{R}^+$ , satisfy  $\lim_{i \rightarrow \infty} \lambda_i = \infty$  and the first (or fundamental) eigenvalue corresponds to a positive eigenfunction. Moreover,  $\lambda_1$  is the only one with a positive eigenfunction. Assuming self-adjointness these features hold for general second order elliptic boundary value problems. With fourth order elliptic eigenvalue problems, such as  $\Delta^2\varphi = \lambda\varphi$ , under appropriate boundary values, some of these features remain but most results related with positivity are lost. In the talk I will address some examples that illustrate the differences between second and fourth order elliptic problems. Partly joint work with Inka Schnieders and Jan Gerdung.

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## **Extrapolation, weights and multiplier theorems in UMD spaces**

Mark Veraar

In this talk we discuss several versions of Rubio de Francia's extrapolation theorem for functions which take values in a UMD Banach function space. This leads to short proofs of some new and known results. In particular we obtain Littlewood-Paley-Rubio de Francia estimates for Banach function spaces with UMD concavifications. We show that the results

can be used to obtain vector-valued versions of multiplier result due to Coifman-Rubio de Francia-Semmes. To cover the case of operator-multipliers we introduce a variant of  $R$ -boundedness.

The talk is based on joint work with Alex Amenta and Emiel Lorist.

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### **When do the regular operators between two Banach lattices form a lattice?**

Tony Wickstead

We investigate when the regular operators from one Banach lattice into another form a vector lattice. We give complete results when the domain is either separable or has an order continuous norm. In these two settings, at least, the lattice operators are given by the Riesz-Kantorovich formulae, in contrast with Elliott's negative result for the general setting.

# Participants

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