Linear algebra 2: exercises for Chapter 3 (Cayley-Hamilton)

Ex. 2.1. What is the remainder when one divides the polynomial $x^5 + x$ by $x^2 + 1$?

Ex. 2.2. Give the minimal polynomial and the characteristic polynomial of the matrices

$$\begin{pmatrix} 2 & -3 & 3 \\ 3 & -4 & 3 \\ 3 & -3 & 2 \end{pmatrix}, \qquad \begin{pmatrix} 0 & -1 & 3 \\ 1 & -2 & 3 \\ 3 & -3 & 2 \end{pmatrix}.$$

Ex. 2.3. Suppose that a 2×2 matrix A has two distinct eigenvalues λ and μ. Show that the image of the matrix $A - \lambda$ is the eigenspace with eigenvalue μ.

Ex. 2.4. Is the matrix $\begin{pmatrix} 0 & 0 & -3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ diagonalizable over $\mathbb{R}$? And over $\mathbb{C}$?

Ex. 2.5. If $f\colon \mathbb{R}^3 \to \mathbb{R}^3$ is the projection on a plane, what is the minimum polynomial of f? What is the minimum polynomial of relection in a plane?

Ex. 2.6. Compute the characteristic polynomial of the matrix

$$A = \begin{pmatrix} 1 & -9 & 4 \\ 1 & -4 & 1 \\ 1 & -7 & 3 \end{pmatrix}.$$

Compute A^3 (use Cayley-Hamilton!)

Ex. 2.7. Let V be the 4 dimensional vector space of polynomial functions $\mathbb{R} \to \mathbb{R}$ of degree at most 3. Let $T\colon V \to V$ be the map that sends a polynomial p to its derivative $T(p) = p'$. Show that T is a linear map. Is T diagonalizable?

Ex. 2.8. For each $\alpha \in \mathbb{R}$, determine the characteristic and minimal polynomials of

$$A_\alpha = \begin{pmatrix} 1-\alpha & \alpha & 0 \\ 2-\alpha & \alpha - 1 & \alpha \\ 0 & 0 & -1 \end{pmatrix}.$$

For which values of α is A_α diagonalizable?