Points, lines, planes and more

1. Lines in the plane, equations and parametrizations

Exercises.

Exercise 1.1. *Compute the inner product of the given vectors v and w.*

- $v = (-2, 5)$ *and* $w = (7, 1)$,
- $v = 2(-3, 2)$ *and* $w = (1, 3) + (-2, 4)$,
- $v = (-3, 4)$ *and* $w = (4, 3)$,
- $v = (-3, 4)$ *and* $w = (8, 6)$,
- $v = (2, -7)$ *and* $w = (x, y)$,
- $v = w = (a, b)$.

Exercise 1.2. *Write the following equations in the form $\langle a, x\rangle = c$, i.e., specify a vector a and a constant c in each case.*

(1) $L_1\colon 2x_1 + 3x_2 = 0$,
(2) $L_2\colon x_2 = 3x_1 - 1$,
(3) $L_3\colon 2(x_1 + x_2) = 3$,
(4) $L_4\colon x_1 - x_2 = 2x_2 - 3$,
(5) $L_5\colon x_1 = 4 - 3x_1$,
(6) $L_6\colon x_1 - x_2 = x_1 + x_2$.
(7) $L_7\colon 6x_1 - 2x_2 = 7$

Exercise 1.3. *Paramerize the lines in Exercise 1.2.*

Exercise 1.4. *Write the following parametrizations in the form $x = tp + q$, i.e., specify vectors p and q in each case.*

(1)
$$M_1\colon \begin{cases} x_1 &= t + 2, \\ x_2 &= -t - 3., \end{cases}$$

(2)
$$M_2\colon \begin{cases} x_1 &= 2t + 1, \\ x_2 &= 6t - 3, \end{cases}$$

(3)
$$M_3\colon \begin{cases} x_1 &= 3t - 1, \\ x_2 &= t, \end{cases}$$

(4)
$$M_4\colon \begin{cases} x_1 &= 3s + 5, \\ x_2 &= 7, \end{cases}$$

(5)
$$M_5\colon \begin{cases} x_1 &= t + 4, \\ x_2 &= t + 4, \end{cases}$$

(6)
$$M_6\colon \begin{cases} x_1 &= 3t, \\ x_2 &= 4t. \end{cases}$$

Exercise 1.5. *Give an equation for lines in Exercise 1.4.*

Exercise 1.6. *True or False? If true, explain why. If false, give a counter example.*

(1) *If a and b are nonzero vectors and $a \neq b$, then the lines given by $\langle a, x\rangle = 0$ and $\langle b, x\rangle = 1$ are not parallel.*
(2) *If a and b are nonzero vectors and the lines given by $\langle a, x\rangle = 0$ and $\langle b, x\rangle = 1$ are parallel, then $a = b$.*
(3) *Two different lines may be given by the same equation.*
(4) *Two different parametrizations may parametrize the same line.*
(5) *The intersection of two lines is either empty or consists of one point.*
(6) *For each vector $v \in \mathbb{R}^2$ we have $0 \cdot v = 0$. (What do the zeroes in this statement refer to?)*

Exercise 1.7. *Which of all the lines in Exercises 1.2 and 1.4 are parallel?*

Exercise 1.8. *Compute the point of intersection of the following lines from Exercises 1.2 and 1.4, if it exists.*

(1) L_1 *and* L_2,
(2) L_2 *and* L_3,
(3) L_5 *and* M_4,
(4) L_4 *and* M_2,
(5) L_2 *and* M_2,
(6) M_2 *and* M_3.

Exercise 1.9. *When you intersect two lines, would you prefer them to be given by a parametrization or by an equation?*

Exercise 1.10. *Given the vectors $p = (p_1, p_2)$ and $q = (q_1, q_2)$ with $p \neq 0$, we let L be the line parametrized by $x = tp + q$. Find a vector a and a constant c (in terms of p_1, p_2, q_1, q_2) such that L is defined by the equation $\langle a, x\rangle = c$.*

Exercise 1.11. *Let a and b be two vectors and c and d two constants. Let L be the line given by $\langle a, x\rangle = c$ and M the line given by $\langle b, x\rangle = d$. Show that M and L are perpendicular if and only if $\langle a, b\rangle = 0$.*

Exercise 1.12. *Give an equation and a parametrization, both written in terms of vectors, of the following lines in $\mathbb{R}^2$.*

(1) *The line through the points $(1, 2)$ and $(3, -5)$.*
(2) *The line through the point $(3, 2)$ that is parallel to the line given by $3x_1 - x_2 = 0$.*
(3) *The line through the point $(3, -2)$ that is perpendicular to the line given by $\langle a, x\rangle = -1$ with $a = (1, -2)$.*
(4) *The line through the point $(1, 4)$ that is parallel to the line parametrized by $x = pt + q$ with $p = (1, 2)$ and $q = (-3, 4)$.*
(5) *The line through the point $(1, 4)$ that is perpendicular to the line parametrized by*

$$\begin{cases} x_1 = 3t - 1, \\ x_2 = 4t + 1. \end{cases}$$

(6) *The perpendicular bisector of the line segment from $(-2, -1)$ to $(4, 3)$.*
(7) *The line obtained by rotating the line given by $\langle (3, 5), x\rangle = 2$ around the origin over $90°$ (so counter clockwise).*
(8) *(*) The line obtained by rotating the line given by $\langle (3, 5), x\rangle = 2$ around the origin over α.*