

Exercises elliptic curves, Fall 2009, week 4

Choose four of the exercises from either paragraph 8 in Cassels' book, or from exercises 1.18, 1.19, 1.20 in the book by Silverman and Tate, or of the following two exercises.

A

Let C be a nonsingular projective curve of genus 0 over the field k and let $P \in C(k)$ be a point. Show that $L(P)$ contains a nonconstant function, say f . Furthermore, show that the map $C(k) \setminus \{P\} \rightarrow k$ that sends R to $f(R)$ is a bijection.

B

In each of the examples (i), (ii), and (iii) of paragraph 8 in Cassels, express the functions x_W, y_W that give the Weierstrass coordinates in terms of the original variables. Show that indeed we have $x_W \in L(2\mathcal{O})$ and $y_W \in L(3\mathcal{O})$.