Choose four exercises from paragraph 11 and 12 in Cassels, exercises 2.4, 2.10, 2.11, 2.12 in Silverman–Tate (avoiding of course near-doubles like 2.12 and 12.1 from Cassels), and the exercise below.

E Let C be an elliptic curve over $\mathbb{Q}_p$ with Weierstrass form

$$y^2 = x^3 + ax^2 + bx + c.$$

As before, we say that the level of a point (x, y) is n if $v_p(x) = -2n$ and $v_p(y) = -3n$, and we let $C^{(n)}(\mathbb{Q}_p)$ denote the set of all points of level at least n. Define the functions $t = x/y$ and $z = 1/y$. Then there is another affine part of C that is given by

$$z = t^3 + at^2z + btz^2 + cz^3.$$

(1) Show that the point 0 corresponds to $(t, z) = (0, 0)$ in this affine part.
(2) Show that on this new affine part negation is given by $-(t, z) = (-t, -z)$.
(3) Show that if $n > 0$, then $C^{(n)}(\mathbb{Q}_p)$ corresponds with

$$\{(t, z) \ : \ v_p(t) \geq n \text{ and } v_p(z) > 0\}.$$

(4) Show that the level of a point $P \in C^{(1)}(\mathbb{Q}_p)$ equals $v_p(t(P))$.
(5) Show that for $(t, z) \in C^{(1)}(\mathbb{Q}_p)$ of level n we have $v_p(z(P)) = 3n$.