GAPS IN ORBITS OF N-EXPANSIONS

JAAP DE JONGE AND COR KRAAIKAMP

Abstract. For $N \in \mathbb{N}$ and $\alpha \in \mathbb{R}$ such that $0 < \alpha \leq \sqrt{N} - 1$, the function $T_\alpha : [\alpha, \alpha+1] \to [\alpha, \alpha+1)$ is defined as $T_\alpha(x) := \frac{N}{x} - d(x)$, where $d : [\alpha, \alpha+1] \to \mathbb{N}$ is defined by $d(x) := \left\lfloor \frac{N}{x} - \alpha \right\rfloor$. For $N \leq 7$ there are α, intervals $(a, b) \subset [\alpha, \alpha+1]$ and $n_0 \in \mathbb{N}$ such that $T_\alpha^n \cap (a, b) = \emptyset$ for all $n \geq n_0$. These *gaps* (a, b) are investigated in the unit square $\Upsilon_\alpha := [\alpha, \alpha+1] \times [\alpha, \alpha+1]$, in which the elements of the *orbits* $T_\alpha^k(x), k = 0, 1, 2, \ldots$ of numbers $x \in [\alpha, \alpha+1]$ are connected by line segments, so as to form a *web graph*. The square Υ_α is divided in *cylinders* Δ_d, according to the value of d in the function T_α. In the case of two cylinders there may be one or no gap on $[\alpha, \alpha+1]$; in the case of three cylinders, there are either none, one, two or three gaps, depending both on N and α; in the case of four cylinders there are usually no gaps, but when they occur, they are extremely wide; in the case of five or more cylinders no gaps occur.

Jaap de Jonge, Delft University of Technology, department of Electrical Engineering, Mathematics and Computer Science, Mekelweg 4, 2628 CD Delft and University of Amsterdam, Korteweg - de Vries Institute for Mathematics, Science park 105 - 107, 1098 XG Amsterdam, The Netherlands

E-mail address: `c.j.dejonge@uva.nl, c.j.dejonge@tudelft.nl`

Cor Kraaikamp, Delft University of Technology, department of Electrical Engineering, Mathematics and Computer Science, Mekelweg 4, 2628 CD Delft, The Netherlands

E-mail address: `C.Kraaikamp@tudelft.nl`

Date: March 6, 2018.
Key words and phrases. Continued fractions, Metric theory.