The typical shapes of the EFT functions for the class of covariant Galileon Lagrangians

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Abstract

One of the major goals in cosmology is explaining the acceleration of the expansion of the universe. To do this, we examine a theory of modified gravity. We look at the covariant Galileon Lagrangian class of models, and model the Effective Field Theory functions for a choice of test parameters by using the tracking solution for the scalar field on which the Galileon Lagragian is based. Next we examine the stability of the theory for a range of values for the tracking parameter by checking for the positivity of the kinetic term and by checking for which parameter sets the speed of sound of the scalar field does not turn imaginary. These checks gave us reasonable parameter spaces, but the exact values which our main reference [1] gives were not included in the space, however, with error bars they are.

Contents

Chapter 1

Introduction

General relativity was introduced by Albert Einstein in 1915, in his paper 'The foundation of general relavity', in which he describes a new way of looking at gravity and the equation that would govern it, named after him: the Einstein equation. Initially, Einstein believed his equation could not be solved analytically, but this was quickly disproven by Karl Schwarzshield. He gave the solution to the Einstein equation in the case of a spherically symmetric universe around a massive object. Soon, others followed with exact solutions to the Einstein equation.

In 1922, Russian physicist Alexander Friedmann introduced the first non-static solutions to Einsteins equation of general relativity. In his paper, he analyses three different scenarios in which the universe either expands monotonically, which covers two of the scenarios, or is periodic. His papers were initially ignored and Einstein deemed his results to be without physical meaning [2]. In contrast, Einstein had assumed that our universe had to be static, and thus was only looking for static solutions. In order to preserve this static behaviour of the universe, Einstein had added the cosmological constant λ to his equations, which could be used to compensate for any force that would cause the universe to expand or contract. This was only a mathematical trick, and not without consequences. Einsteins static solution was very unstable, and any perturbation would cause the universe to start expanding or contracting. Even with this in mind, Einstein held on to his belief for over eight years, until new evidence appeared.

It was Edwin Hubble in 1929 who was able to give proof that our universe was, in fact, expanding. He observed distant galaxies and found that they were moving away from us. Upon reading this, Einstein erased the cosmological constant from his equation and called it 'his biggest blunder'.

Nevertheless, the need for cosmological constant arose again after it became clear that the expansion of the universe has been accelerating, and it has been doing so since about 5 or 6 billion years after the Big Bang. The Einstein equation however, predicts that the expansion should slow down, since the force of gravity is supposed to take over. Now the cosmological constant was used to explain this phenomenon, rather than to keep the universe static. The physical interpretation is that it represents a mysterious force called dark energy, which is speeding up the expansion. Naturally, not all physicists are satisfied with this explanation. No one knows what dark energy is exactly and where it comes from, thus various groups have been searching for alternative theories, or theories that explain what dark energy is.

In this research, we will be looking at one the theories of modified gravity. These theories, as the name implies, modify the theory of gravity in order to explain the accelerated expansion. In particular, we will be studying the theory of Covariant Galileon Lagrangians. This theory introduces an extra field to the universe, which translates into an extra degree of freedom, which could modify gravity in such a way that it eliminates the need for dark energy. This cannot be done in any manner, thus we will focus on constraining it such that it provides us with stable solutions. However, in order understand what this means, we will need to look a bit more into what Einsteins theory of general relativity actually consists of.

Chapter 2

The theory of general relativity

2.1 Introduction to four-dimensional spacetime

Generally, physical calculations are done using Newtonian mechanics, in which gravity is a force, very similar to for example electromagnetic force. However, the theory of general relativity proposes that gravity is fundamentally different from other forces, since it can be seen as a result of the curvature of spacetime.

The first step is viewing the universe as a four dimensional manifold, called the spacetime manifold. A manifold M is a space which is locally homeomorphic to a linear space, and in our case our manifold will be locally homeomorphic to $\mathbb{R}^n$. In exact terms, associated with M we have charts $\{(U_\alpha, \phi_\alpha)\}$ in which $U_\alpha \subset M$ are open and cover M, and $\phi_\alpha : U_\alpha \to \mathbb{R}^n$ are homeomorphisms. The collection of charts is called an atlas.
The spacetime manifold is a differentiable manifold, which means that we have an extra requirement. Let $(U_\alpha, \phi_\alpha), (U_\beta, \phi_\beta)$ be any two charts, and let us look at the images of their intersection, so $A = \phi_\alpha(U_\alpha \cap U_\beta)$ and $B = \phi_\beta(U_\alpha \cap U_\beta)$. Then we can define a transition map $\phi_{\alpha\beta} : A \to B$ by setting $\phi_{\alpha\beta} = \phi_\beta \circ \phi_\alpha^{-1}\big|_A$. For our manifold to be a differential manifold, all transition maps need to be differentiable.

This structure is very useful, since even though the spacetime manifold is quite an abstract space without properties like flatness, which is how we do think about the world around us, we can define operations like differentiation on it using the transition maps. This allows us to do calculus as we're used to on manifolds. We can't however always equate the manifold to R^n, since this only works on very small scales. On larger scales, we want to be able to look at the curved structure of our manifold, which is where the metric tensor comes in.

The goal cosmologists are working towards is finding an expression for this metric which accurately describes our universe. We do this by finding the appropriate action, minimizing it, and by doing so extracting the equations of motion of the metric. This will be a set of second order differential equations which will give us the metric by solving them. However, before we get there, we will need the necessary definitions and tools.

2.2 The metric tensor

The structure of spacetime can be collected in the metric $g_{\mu\nu}$, which is defined as a symmetric $(0,2)$-tensor, usually with a non-vanishing determinant. Simply said, the metric gives you the distance between any two points on your manifold. Since it's a $(2,0)$-tensor, we can write it like a matrix, such that in, for example two dimensions, the distance between the points (x, y) and $(x + dx, y + dy$ is given by:

$$\begin{pmatrix} dx & dy \end{pmatrix} \begin{pmatrix} g_{xx} & g_{xy} \\ g_{xy} & g_{yy} \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix} = g_{xx}dx^2 + 2g_{xy}dx^2dy^2 + g_{yy}dy^2. \tag{2.1}$$

If we choose the identity matrix I for our metric, then the metric reduces to the inner product on Euclidean space. This is not surprising, since the inner product is used to calculate distances on Euclidean space, thus the metric tensor can be seen as a generalisation of it.
If we expand our two-dimensional example to n dimensions, we will write a line element of the manifold as:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu. \tag{2.2}$$

Again, in flat space, the metric is given by the identity matrix, thus we have:

$$ds^2 = dx_1^2 + ... + dx_n^2. \tag{2.3}$$

Fortunately, we recognize this as the Pythagorean theorem, and it is of course exactly what we expected. It would have been cause for worry if calculating distances with the metric gave a different result than calculating it in the regular way.
To give a less trivial example, we can also have a look at the metric of the $2-$sphere. The $2-$sphere is given by $S^2 = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$. This is clearly a curved surface, so the associated metric will be non-Euclidean. It is given by:

$$g = \begin{pmatrix} 1 & 0 \\ 0 & \sin^2\theta \end{pmatrix}. \tag{2.4}$$

In this expression we switched from Cartesian coordinates to spherical coordinates θ and ϕ. The line element is easily found, and given by:

$$ds^2 = d\theta^2 + \sin^2\theta d\phi^2. \tag{2.5}$$

To finish the example, let's look at path between the points $(\theta, \phi) = (0, \pi/2)$ which is a point on the equator, and $(\pi/2, \pi)$, which we arrive at by travelling across the equator halfway round the sphere. We already know that the answer should be π, and calculating it gives exactly that:

$$L = \int_0^\pi d\theta, \tag{2.6}$$

$$= \pi. \tag{2.7}$$

2.3 Covariant derivatives

In order to do any meaningful calculations on the vectors on our spacetime manifold, we will need the notion of a derivative. It may seem easiest to just use directional derivatives as we know them, but it turns out that they are not suitable. We want our derivative to be independent of basis, thus, as we change our basis, our derivative should transform in the same way. Let μ, ν be vectors in the basis for our manifold $\mathcal{M}$ and μ', ν' be vectors from another basis, then the covariant derivative in the direction μ' of the vector $V^{\nu'}$ must obey the transformation law:

$$\nabla_{\mu'} V^{\nu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\nu'}}{\partial x^{\nu}} \nabla_{\mu} V^{\nu}. \tag{2.8}$$

On flat space, we naturally want the covariant derivative to reduce to regular partial derivatives, thus it makes sense to define the covariant derivative as a partial derivative plus some correction term that ensures that the transformation law is followed, but vanishes when the manifold is flat. Inuitively, these correction terms correct for the curvature of the manifold. They are called Christoffel symbols and are denoted by Γ. The covariant derivative in terms of partial derivatives and Christoffel symbols is given by:

$$\nabla_{\mu} V^{\nu} = \partial_{\mu} V^{\nu} + \Gamma^{\nu}_{\mu\lambda} V^{\lambda}. \tag{2.9}$$

From this expression we see that Γ has to be a $n \times n$ matrix, with n being the dimension of our manifold, so in spacetime it will be a 4×4 matrix. Since the Christoffel symbols account for the curvature, we will want to express them in terms of the metric. This expression is given by:

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma} (\partial_{\mu} g_{\nu\sigma} + \partial_{\nu} g_{\sigma\mu} - \partial_{\sigma} g_{\mu\nu}). \tag{2.10}$$

Here, $g^{\lambda\sigma}$ is the inverse metric, which is defined by $g^{\lambda\sigma} g_{\sigma\mu} = \delta^{\lambda}_{\mu}$.
It's not hard to see that the Christoffel symbols do indeed vanish when we choose g to be the flat metric (see eq. 2.3), which makes sure that our covariant derivatives reduce to partial derivatives in flat space.

2.4 The Riemann tensor

Thus far I've spoken quite a bit about curvature and that the spacetime manifold curves in respond to matter, but we don't have a good way of quantifying this curvature yet. Before doing this, we are going to need the concept of parallel transport. Let's call our manifold M, and define a parametrized curve $\alpha : [a, b] \to M$, $a, b \in \mathbb{R}$ and $b > a$. We want a notion of parallel transporting a vector from $\alpha(a)$ to $\alpha(b)$.
In flat space, we would just move our vector along the curve while keeping its components constant. In curved space however, there is no one way of defining one basis that can be used for vectors at any point of M, since we are working with a localized coordinate system. This means that a vector on a point is expressed in the local basis of the tangent space to M at that point. Thus, we define the notion of parallel transport using the covariant derivative. We define the vector field $\mathbf{V}(t) = (\alpha(t), V(t))$ with

$V(t)$ being the vector that is being transported on the point $\alpha(t)$. We speak of parallel transport along α if $\nabla_\mu \mathbf{V}^\mu = 0$. On flat space, where the covariant derivative reduces to partial derivatives, this simply means $\frac{d}{dt}\mathbf{V} = 0$, which is the same as simply saying that the components do not depend on time, just as we expected.

Now that we have a notion of parallel transport, we can use it to quantify curvature. The way we do this is by using the Riemann tensor. To give some intuition, imagine a parallelogram defined by vectors A^μ and B^ν on a flat surface, and any vector V^σ in one of the corners. If we would move V^σ along this parallelogram, it would of course remain unchanged. However, things get more complicated if we imagine our parallelogram to be lying on a curved surface, and parallel transport V^σ around the parallelogram, since after completing the loop, the vector will have changed direction. Making our parallogram infinitely small, this change is given by:

$$\delta V^\rho = R^\rho_{\sigma\mu\nu} V^\sigma A^\mu B^\nu,$$

in which $R^\rho_{\sigma\mu\nu}$ is the Riemann tensor. It is given by:

$$R^\rho_{\sigma\mu\nu} = \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma}. \quad (2.11)$$

From this Riemann tensor we can construct the Ricci tensor and Ricci scalar, which will ultimately appear in equations of motion of the metric. The Ricci tensor is obtained by contracting the Riemann tensor in the following way:

$$R_{\mu\nu} = R^\lambda_{\mu\lambda\nu}. \quad (2.12)$$

It is worth noting that any other contraction of the Riemann tensor either vanishes or is related to the Ricci tensor, thus making this the only independent contraction we can make. Tracing the Ricci tensor gives the Ricci scalar:

$$R = g^{\mu\nu} R_{\mu\nu}. \quad (2.13)$$

With the Ricci tensor and scalar in place, we have all the necessary tools to definine the action from which the equations of motion will follow.

2.5 The Einstein-Hilbert action

An action S in the classical sense is a physical concept of the form:

$$S = \int L \, dt, \quad (2.14)$$

in which L is a quantity called the Lagrangian. In classical mechanics, the Lagrangian is given by $L = T - V$, with V being the potential energy and T being the kinetic energy of a particle. Setting the variation δS to 0 gives us the equations of motion which govern the path of a particle.

We are however, not interested in the equations of motion of a particle, but in the equations which govern the metric $g_{\mu\nu}$. For this we use a field-theoretical action:

$$S = \int \mathcal{L} \, dx^4, \quad (2.15)$$

which integrates the Lagrangian density L over all of spacetime. Whereas the classical action applies to discrete particles, this field-theoretical action is the analogue which applies to fields or other continuous quantities. The harmonic oscillator has only one dimension, thus solving the action gives us one equation of motion. For any variable which you add, you will get an additional equation, so the number of equations obtained by solving the action equals the number of free variables. This will be of importance later in the research.

The action which describes our universe is called the Einstein-Hilbert action, and is given by:

$$S = \int \sqrt{-g}[\frac{1}{16\pi G}R + \mathcal{L}_M]dx^4, \tag{2.16}$$

in which $\mathcal{L}_M$ is the matter Lagrangian, determined by the matter- and energy densities of our universe, $g = \det g_{\mu\nu}$, and G is the gravitational constant.

Solving for $\delta S = 0$ gives us the Einstein equation [3], given by:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{2.17}$$

Here we introduce the tensor $T_{\mu\nu}$, which is called the stress-energy tensor, and describes the energy densities and energy flux in our universe. For example in a vacuum universe, the matter Lagrangian and thus the stress-energy tensor vanish.

Chapter 3

Covariant Galileon Lagrangians

Since the Einstein equation as is does not explain the accelerated expansion of the universe, there are several ways to alter it. By adding different terms to the Einstein-Hilbert action, we can change the way the metric behaves. However, we can't do this at complete random. In general, we want to preserve the independence of coordinates of the Einstein-Hilbert action. This means that we do not want the coordinate system that we choose to influence our final result. Therefore, we can't add any terms dependent on our spatial coordinates or our time coordinate.
We can however, choose to break one of these symmetries. We are going break our time symmetry by introducing a new degree of freedom, namely the ϕ-field. We do this by what is called 3+1 formalism, which decomposes the spacetime manifold into spacelike hypersurfaces which vary with a time coordinate. By choosing a time coordinate, we of course break our time independence, which gives us in return the extra degree of freedom. Formally, we will do this as follows:

Let $\mathcal{M}$ be our spacetime manifold, then we speak of a foliation if there exists a smooth scalar field $\phi : \mathcal{M} \to \mathbb{R}$ such that a hypersurface Σ_t in our foliation is given by:

$$\Sigma_t = \{p \in \mathcal{M}, \phi(p) = t\}. \tag{3.1}$$

Intuitively, one could see the hypersurfaces as slices of the spacetime manifold at a constant time, however, what constant time exactly means is dependent on our choice of ϕ.

3.1 The background field and the tracker solution

At large scales, the universe is homogenous and isotropic, meaning that matter is distributed evenly over the whole universe. Moreover, the universe we will consider is flat, which means that it is described by the Friedmann-Lemaître-Robertson-Walker metric, given by:

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \tag{3.2}$$

in which $a(t)$ is the scale factor which governs the spatial expansion of the universe. While we can't measure the scale factor, we can measure the Hubble parameter, given by:

$$H = \frac{\dot{a}(t)}{a(t)}, \tag{3.3}$$

in which $\dot{a}(t)$ represents the time derivative of $a(t)$. The present day Hubble factor, H_0, is estimated at 67.3 (km/s)/Mpc [7], which is the value we will subsequently use in this research.
Since we are considering the universe at large scales only, we will also decompose the scalar field into the background field and local perturbations such that $\phi = \phi_0 + \delta\phi$. In the remaining part of this research, we are only interested in ϕ_0.

It turns out that the initial conditions of the background field do not influence the time evolution in the sense that all fields converge to the same solution [4]. This is called the tracker solution and it is approached before the accelerated expansion which we try to solve. Thus, it is reasonable to use it instead of trying to solve the field ourselves. The tracker solution is characterised by:

$$H\dot{\varphi}_0 = \xi H_0^2, \tag{3.4}$$

in which φ is the dimensionless scalar field $\varphi_0 = \phi_0/m_0$, and m_0 is the Planck mass. For the evolution of H we will make use of the following function $E = \frac{H}{H_0}$ given by:

$$E(a) = \sqrt{\frac{1}{2}\left(\Omega_{r0}a^{-4} + \Omega_{m0}a^{-3} + \sqrt{(\Omega_{r0}a^{-4} + \Omega_{m0}a^{-3})^2 + 4\Omega_{\phi 0}}\right)}, \tag{3.5}$$

[1], in which Ω_{r0} and Ω_{m0} are the cosmological parameters governing respectively the background densities of radiation, and baryonic and cold dark matter, and $\Omega_{\phi 0}$ is defined as $\Omega_{\phi 0} = 1 - \Omega_{r0} - \Omega_{m0}$. Formally, the cosmological parameter for neutrino density $\Omega_{\nu 0}$ should be included in $E(a)$ as well but we set it to zero, since its influence is neglegible in comparison to the other cosmological parameters. We will also set $\Omega_{r0} = 10^{-4}$ for the remainder of the research. This is the value as measured by cosmic background experiments, but the exact value is not very important. While radiation dominated in the early universe, it has become also negligible nowadays.

3.2 The covariant Galileon action

The covariant Galileon model uses the scalar field ϕ (the complete ϕ, not only the background part) to add extra terms to the Einstein-Hilbert action in the most complete manner possible. It turns out there are only five ways to make compose a Lagrangian density out of this scalar field. These five expressions are called the covariant Galileon

Lagrangians. Defining $M^3 = m_0 H_0^2$, the five Lagrangian densitities are given by [1]:

$$L_1 = M^3\varphi, \tag{3.6}$$
$$L_2 = \nabla_\mu\varphi\nabla^\mu\varphi, \tag{3.7}$$
$$L_3 = \frac{2}{M^3}\Box\varphi\nabla_\mu\varphi\nabla^\mu\varphi, \tag{3.8}$$
$$L_4 = \frac{1}{M^6}\nabla_\mu\varphi\nabla^\mu\varphi\big[2(\Box\varphi)^2 - 2(\nabla_\mu\nabla_\nu\varphi)(\nabla^\mu\nabla^\nu\varphi) - R\nabla_\mu\varphi\nabla^\mu\varphi/2\big], \tag{3.9}$$
$$\begin{aligned} L_5 = {}& \frac{1}{M^9}\nabla_\mu\varphi\nabla^\mu\varphi\big[(\Box\varphi)^3 - 3(\Box\varphi)(\nabla_\mu\nabla_\nu\varphi)(\nabla^\mu\nabla^\nu\varphi) \\ &+ 2(\nabla_\mu\nabla^\nu\varphi)(\nabla_\nu\nabla^\rho\varphi)(\nabla_\rho\nabla^\mu\varphi) - 6(\nabla_\mu\varphi)(\nabla^\mu\nabla^\nu\varphi)(\nabla^\rho\varphi)G_{\nu\rho}\big]. \end{aligned} \tag{3.10}$$

In this definition instead of the ϕ, again the rescaled field $\varphi = \phi/m_0$ is used.
The accompanying action is given by:

$$S = \int dx^4\sqrt{-g}\Big[\frac{R}{16\pi G} - \frac{1}{2}\sum_{i=1}^{5} c_i L_i - \mathcal{L}_M\Big], \tag{3.11}$$

in which $c_i \in \mathbb{R}$ are coefficients to give a weight to each of the covariant Galileon Lagrangians.

The c_i's are however not all independent. Since L_1 is not physically interesting, we will set $c_1 = 0$. Furthermore, the c_i's and ϕ are subject to scaling degeneracy. This means that they are invariant under to following transformation for any $B \in \mathbb{R}$:

$$c_i \to c_i/B^i, \text{ for } i = 2,3,4,5, \tag{3.12}$$
$$\phi \to \phi B. \tag{3.13}$$

[1]. Thus, we are allowed to fix one of the parameters, as long as we do not change signs. Since c_2 is constrained such that it will always be negative [1], we set $c_2 = -1$. From now on, we wil speak of the cubic model, or L_3, when we choose $c_4 = c_5 = 0$, the quartic model or L_4 when we set $c_5 = 0$ and the quintic model or L_5 will refer to the full model.

By solving the action we get two equations of motion, namely one for each field. The equation of motion of the background scalar field ϕ_0 gives us the following constraint on the relation between c_i and ξ:

$$c_2\xi^2 + 6c_3\xi^3 + 18c_4\xi^4 + 15c_5\xi^5 = 0. \tag{3.14}$$

Choosing to use the FLRW-metric as in equation 3.2 means that the Friedmann equations are applicable to our model. Solving them for the tracker solution gives the relation between the Galileon parameters c_2 to c_5 and the cosmic parameters, as given by :

$$1 - \Omega_{r0} - \Omega_{m0} = \frac{1}{6}c_2\xi^2 + 2c_3\xi^3 + \frac{15}{2}c_4\xi^4 + 7c_5\xi^5 \tag{3.15}$$

[1]. Since Ω_{r0} is negligibly small compared to Ω_{m0} (approximately 10^{-4} and 0.3), we will set it to zero.

These two equations allow us the remove some dependencies. In all cases we have $c_2 = -1$, as mentioned before, which reduces our free parameters by one. We will now do some work which is prerequisite for working with the models and consider the three cases L_3, L_4, and L_5 seperately to see how we can most effectively reduce their free parameters using equations 3.14 and 3.15.

In the L_3 case, we have $c_4 = c_5 = 0$, so we are left with three parameters (Ω_{m0}, ξ, and c_3) and two equations, which allows us to rewrite both ξ and c_3 in terms of Ω_{m0}. This gives us:

$$c_3 = \frac{1}{6\sqrt{6(1-\Omega_{m0})}}, \tag{3.16}$$

$$\xi = \sqrt{6(1-\Omega_{m0})}. \tag{3.17}$$

In the L_4 case, we only have $c_5 = 0$, which leaves with the parameters Ω_{m0}, ξ, c_3, c_4. Now, we will use equation 3.14 to rewrite c_3 in terms of the other parameters, and then substitute that expression in equation 3.15 to express ξ in Ω_{m0} and c_4. This gives us:

$$c_3 = \frac{1}{6\xi} - 3\xi c_4, \tag{3.18}$$

$$\xi = \frac{1}{6}\sqrt{\frac{\sqrt{5}\sqrt{-432c_4\Omega_{m0} + 432c_4 + 5} - 5}{c_4}}. \tag{3.19}$$

In the L_5 case, we will have to think carefully about what parameters to rewrite. Now we use the full form of equation 3.15, which is a quintic polynomial and thus there exists no standard solution. Thus, we choose to rewrite c_5 using equation 3.14 and express c_4 in terms of ξ, c_3, and Ω_{m0}, in order to avoid solving the quintic polynomial. This would also force us to choose between solutions, which would make it unnecessarily complicated. Thus we get:

$$c_5 = \frac{1}{15\xi}(-18c_4\xi^2 - 6c_3\xi + 1), \tag{3.20}$$

$$c_4 = \frac{10\Omega_{m0} - 8\xi^3 c_3 + 3\xi^2 - 10}{9\xi^4}. \tag{3.21}$$

In conclusion, we will have only Ω_{m0} as free parameter for L_3, Ω_{m0} and c_4 as free parameters for L_4, and Ω_{m0}, ξ, and c_3 as free parameters for L_5.

3.3 Deriving the EFT functions

The covariant Galileon Lagrangians as we just gave them, are however not written in the language that is useful to us. Generally, we want to work with what is called the complete EFT action, in which EFT stands for Effective Field Theory. This complete action contains terms of every quantity that is independent of our spatial coordinates, but can be dependent on time. Thus it is, as the name implies, the most complete action we can think of. The complete EFT action is given by:

$$\begin{aligned} S^{EFT} = \int d^4x\sqrt{-g}\Big[&\frac{M_0}{2}(1+\Omega)R + \Lambda - c\delta g^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{M}_1^3}{2}\delta g^{00}\delta K \\ &- \frac{\bar{M}_2^2}{2}(\delta K)^2 - \frac{\bar{M}_3^2}{2}\delta K^\mu_\nu \delta K^\nu_\mu + \hat{m}^2\delta N\delta\mathcal{R} + m_2^2(g^{\mu\nu} + n^\mu n^\nu \partial_\mu g^{00}\partial_\nu g^{00}) \\ &+ \lambda_1(\delta\mathcal{R})^2 + \lambda_2\delta\mathcal{R}^\mu_\nu\delta\mathcal{R}^\nu_\mu + \frac{\bar{m}_5}{2}\delta\mathcal{R}\delta K + \lambda_3\delta\mathcal{R}h^{\mu\nu}\nabla_\mu\partial_\nu g^{00} \\ &+ \lambda_4 h^{\mu\nu}\partial_\mu g^{00}\nabla^2\partial_\nu g^{00} + \lambda_5 h^{\mu\nu}\nabla_\mu\mathcal{R}\nabla_\nu\mathcal{R} + \lambda_6 h^{\mu\nu}\nabla_\mu R_{ij}\nabla_\nu R^{ij} \\ &+ \lambda_7 h^{\mu\nu}\partial_\mu g^{00}\nabla^4\partial_\nu g^{00} + \lambda_8 h^{\mu\nu}\nabla^2\mathcal{R}\nabla_\mu\partial_\nu g^{00}\Big]. \end{aligned} \tag{3.22}$$

The coefficients are called the EFT functions and they are only time dependent. Our first goal will be to express these for the case of the covariant Galileon Lagrangians. This will allow us to calculate the stability of a model, which is expressed in these EFT functions. In the covariant Galileon model, the only EFT function which will not vanish are M_4^2, $\bar{M}_1^3$, $\bar{M}_2^2$, $\bar{M}_3^2$, $\hat{M}^2$, Ω, c, and Λ, of which Λ is not important to our research since it does not affect stability. We will calculate the others for the three cases with free coefficients: L_3, L_4, and L_5. For each of these cases, we will make use of the mapping for a general Galileon Lagrangian [5], which will allow us to explicitly give the EFT functions in terms of the first and higher derivatives of the background field ϕ (we will drop the subscript 0 from now on to lighten notation), the tracker solution E and the tracker parameter ξ, the constants c_3, c_4, and c_5 associated with each Lagrangian and the present day Hubble constant H_0.

For plotting, it is useful to rescale the EFT functions such that they are dimensionless, which we will be done as follows:

$$\gamma_1 = M_2^4/(m_0^2 H_0^2), \tag{3.23}$$
$$\gamma_2 = \bar{M}_1^3/(m_0^2 H_0), \tag{3.24}$$
$$\gamma_3 = \bar{M}_2^2/m_0^2, \tag{3.25}$$
$$\gamma_4 = \bar{M}_3^2/m_0^2, \tag{3.26}$$
$$\gamma_5 = \hat{M}^2/m_0^2. \tag{3.27}$$

In the next sections we will give Ω, c, and the γ functions for L_3, L_4, and L_5. For some functions we have chosen to rewrite them explictly terms of the tracker solution E and the tracking parameter ξ, while for others (mainly those with longer expressions) we have kept them in terms of $\dot{\phi}$ and its derivatives. The expressions for these are found by combining equations 3.4 and 3.5. This gives:

$$\dot{\varphi} = \frac{\xi H_0}{E(a)}, \tag{3.28}$$

$$\ddot{\varphi} = \frac{-aH_0^2\xi E'(a)}{E(a)}, \tag{3.29}$$

$$\dddot{\varphi} = -aH_0^3\xi\big[E'(a) - \frac{a(E'(a))^2}{E(a)} + aE''(a)\big], \tag{3.30}$$

in which $E'(a)$ denotes the a-derivative of $E(a)$.

3.3.1 Cubic Lagrangian: L_3

A general cubic Galileon Lagrangian (so not necessarily covariant) can be written in the form:

$$L_3 = G_3(\phi, X)\Box\phi, \tag{3.31}$$

in which $X = \nabla^\mu\phi\nabla_\mu\phi$ is the kinetic term. Note that since the background scalar field is only time-dependent, we have $\nabla^\mu\phi\nabla_\mu\phi = \dot{\phi}^2$. Comparing with the expression for the covariant cubic Lagrangian in equation 3.8 gives us $G_3(\phi, X) = \frac{1}{2}c_3\frac{2}{M^3}X$. The non-zero EFT functions are given by:

$$M_2^4(t) = G_{3X}\frac{\dot{\phi}^2}{2}(\ddot{\phi} + 3H\dot{\phi}), \tag{3.32}$$
$$\bar{M}_1^3(t) = -2G_{3X}\dot{\phi}^3, \tag{3.33}$$
$$c(t) = \dot{\phi}^2 G_{3X}(3H\dot{\phi} - \ddot{\phi}), \tag{3.34}$$
$$\Omega = 1, \tag{3.35}$$

in which G_{3X} denotes the X-derivative of G_3. Thus, we have $G_{3X} = \frac{1}{2}c_3\frac{2}{M^3}$. Plugging in our expressions for G_3, G_{3X}, and ϕ, and rescaling gives:

$$\gamma_1 = \frac{1}{2}c_3\frac{\xi^3}{E^2(a)}\left[-\frac{a}{E(a)}\frac{d}{da}E(a) + 3\right], \tag{3.36}$$
$$\gamma_2 = -\frac{1}{2}c_3\frac{4\xi^3}{E^3(a)}. \tag{3.37}$$

Of course Ω remains unchanged, and for c we get:

$$c(t) = \frac{c_3 m_0^2}{H_0}(3H_0E\dot{\varphi}^3 - \dot{\varphi}^2\ddot{\varphi}). \tag{3.38}$$

3.3.2 Quartic Lagrangian: L_4

A general quartic Galileon Lagrangian is written in the form:

$$L_4 = G_4(\phi, X)R - 2G_{4X}(\phi, X)((\Box\phi)^2 - \nabla^\mu\nabla^\nu\phi\nabla_\mu\nabla_\nu\phi), \tag{3.39}$$

which gives us $G_4 = -\frac{1}{2}c_4\frac{1}{M^6}X^2$, $G_{4X} = -\frac{1}{2}c_4\frac{1}{M^6}X$, and $G_{4XX} = -\frac{1}{2}c_4\frac{1}{M^6}$ by comparing with equation 3.9. The non-zero EFT functions are given by:

$$M_2^4(t) = G_{4X}(-2\dot{H}\dot{\phi}^2 - H\dot{\phi}\ddot{\phi} - \ddot{\phi}^2) + G_{4XX}(18H^2\dot{\phi}^2 + 2\dot{\phi}^2 + 4H\ddot{\phi}\dot{\phi}^3), \tag{3.40}$$
$$\bar{M}_1^3(t) = G_{4X}(4\dot{\phi}\ddot{\phi} + 8H\dot{\phi}^2) - 16HG_{4XX}\dot{\phi}^4, \tag{3.41}$$
$$\bar{M}_2^2(t) = 4G_{4X}\dot{\phi}^2, \tag{3.42}$$
$$\bar{M}_3^2(t) = -4G_{4X}\dot{\phi}^2, \tag{3.43}$$
$$\hat{M}^2(t) = 2G_{4X}\dot{\phi}^2, \tag{3.44}$$
$$\begin{aligned} c(t) &= G_{4X}(2\ddot{\phi}_0^2 + 2\dot{\phi}_0\dddot{\phi}_0 + 4\dot{H}\dot{\phi}_0^2 + 2H\dot{\phi}_0\ddot{\phi}_0 - 6H^2\dot{\phi}_0^2) \\ &+ G_{4XX}(12H^2\dot{\phi}_0^4 - 8H\dot{\phi}_0^3\ddot{\phi}_0 - 4\dot{\phi}_0^2\ddot{\phi}_0^2), \end{aligned} \tag{3.45}$$
$$\Omega(t) = \frac{1}{m_0^2}G_4 - 1. \tag{3.46}$$

Plugging in our expressions and rescaling gives:

$$\gamma_1 = -\frac{1}{2}c_4\frac{\xi^4}{E^2(a)}\big[19 - \frac{3a}{E(a)}\frac{d}{da}E(a) - \frac{a^2}{E^2(a)}\big[\frac{d}{da}E(a)\big]^2\big], \tag{3.47}$$

$$\gamma_2 = \frac{1}{2}c_4\frac{\xi^4}{E^3(a)}\big[4\frac{d}{da}E(a) + 24\big], \tag{3.48}$$

$$\gamma_3 = -\frac{1}{2}c_4\frac{4\xi^4}{E^4(a)}, \tag{3.49}$$

$$\gamma_4 = \frac{1}{2}c_4\frac{4\xi^4}{E^4(a)}, \tag{3.50}$$

$$\gamma_5 = -\frac{1}{2}c_4\frac{2\xi^4}{E^4(a)}, \tag{3.51}$$

for the γ functions, and Ω en c are given by:

$$c(t) = -\frac{c_4}{m_0^2 H_0^4}(-2\dot{\varphi}^2\ddot{\varphi}^2 + 2\dot{\varphi}^3\dddot{\varphi} + 4\dot{H}\dot{\varphi}^4 - 6H\dot{\varphi}^3\ddot{\varphi} + 6H^2\dot{\varphi}^4), \tag{3.52}$$

$$\Omega(t) = -\frac{c_4}{H_0^4}\dot{\varphi}^4 - 1. \tag{3.53}$$

3.3.3 Quintic Lagrangian: L_5

A general quintic Galileon Lagrangian is written in the form:

$$\begin{aligned} L_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi + \frac{1}{2}G_{5X}(\phi, X)\big[(\Box\phi)^3 - 3\Box\phi\nabla^\mu\nabla^\nu\phi\nabla_\mu\nabla_\nu\phi \\ + 2\nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\sigma\phi\nabla_\sigma\nabla^\nu\phi\big], \end{aligned} \tag{3.54}$$

in which $G_{\mu\nu}$ is the Einstein tensor defined as $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$.
Comparing with equation 3.10 gives us $G_5 = -\frac{3}{4}c_5X^2\frac{1}{M^9}$. This time, we will not explicitly give the EFT functions in terms of ξ, c_5, and E, as we did before, but we express it in the dimensionless scalar field φ, H_0, E, and c_5. It is possible to substitute our expressions for φ and its derivatives into these functions, but they get very long and it does not add any insight.

The non-zero EFT functions for L_5 are given by:

$$
\begin{aligned}
M_2^4(t) &= -\frac{1}{2}H^2 G_{5X}\dot{\phi}^3 + \frac{1}{4}m_0^2\dot{\Omega} + \frac{1}{2}Hm_0^2(1+\Omega) \\
&\quad - \frac{3}{4}Hm_0^2\dot{\Omega} + 6G_{5XX}H^3\dot{\phi}^5 - \frac{3}{2}H^3 G_{5X}\dot{\phi}^3, \quad (3.55)\\
\bar{M}_1^3(t) &= -m_0^2\dot{\Omega} - 4H^2\dot{\phi}^5 G_{5XX} + 6H^2\dot{\phi}^3 G_{5X}, \quad (3.56)\\
\bar{M}_2^2(t) &= -\frac{1}{2}G_{5X}\dot{\phi}^2\ddot{\phi} + \frac{1}{2}HG_{5X}\dot{\phi}^3, \quad (3.57)\\
\bar{M}_3^2(t) &= \frac{1}{2}G_{5X}\dot{\phi}^2\ddot{\phi} - \frac{1}{2}HG_{5X}\dot{\phi}^3, \quad (3.58)\\
\hat{M}^2(t) &= -G_{5X}\dot{\phi}^2\ddot{\phi} + HG_{5X}\dot{\phi}^3, \quad (3.59)\\
\Omega(t) &= \frac{2}{m_0^2}G_{5X}\ddot{\phi}\dot{\phi}^2 - 1, \quad (3.60)\\
c(t) &= \frac{1}{2}\dot{\bar{\mathcal{F}}} + \frac{3}{2}Hm_0^2\dot{\Omega} - 3H^3\dot{\phi}^3 G_{5X} + 2H^3\dot{\phi}^5 G_{5XX}. \quad (3.61)
\end{aligned}
$$

For $c(t)$ we have used $\bar{\mathcal{F}}$ to shorten our expression, it is given by:

$$
\bar{\mathcal{F}} = 2H^2 G_{5X}\dot{\phi}^3 - m_0^2\dot{\Omega} - 2Hm_0^2(1-\Omega). \quad (3.62)
$$

Using our expression for G_5 and rescaling gives us:

$$
\begin{aligned}
\gamma_1 &= \frac{3c_5}{5H_0^5}\left[\frac{E^2}{H_0}\dot{\varphi}^5 + \frac{1}{H_0^3}\dot{\varphi}^3(4\ddot{\varphi}^2 + \dot{\varphi}\dddot{\varphi}) + \frac{2E}{H_0^2}\dot{\varphi}^4\ddot{\varphi} - 12E^3\dot{\varphi}^5 + \frac{3E^3}{2}\dot{\varphi}^5\right], \quad (3.63)\\
\gamma_2 &= -\frac{3c_5}{H_0^7}(4\dot{\varphi}^3\ddot{\varphi}^2 + \dot{\varphi}^4\dddot{\varphi}) + \frac{E^2 c_5}{H_0^5}\dot{\varphi}^5, \quad (3.64)\\
\gamma_3 &= \frac{3c_5}{H_0^5}\left[\frac{\dot{\varphi}^4\dot{\varphi}}{H_0} - E\dot{\varphi}^5\right], \quad (3.65)\\
\gamma_4 &= -\frac{3c_5}{H_0^5}\left[\frac{\dot{\varphi}^4\dot{\varphi}}{H_0} - E\dot{\varphi}^5\right], \quad (3.66)\\
\gamma_5 &= \frac{6c_5}{H_0^5}\left[\frac{\dot{\varphi}^4\dot{\varphi}}{H_0} - E\dot{\varphi}^5\right], \quad (3.67)
\end{aligned}
$$

and for Ω and c we get:

$$
\begin{aligned}
\Omega(t) &= \frac{3}{H0^6}c_5\dot{\phi}^4\ddot{\phi} - 1, \quad (3.68)\\
c(t) &= -\frac{3}{4H_0^6}c_5\dot{\varphi}^4(4HH_0\dot{\varphi} + 2H^2\ddot{\varphi}) - \frac{1}{2}\dot{\Omega} \\
&\quad - \dot{H}(1+\Omega) + H\dot{\Omega} - \left(\frac{9H^3}{H_0^6} + \frac{9}{4H_0^6}\right)c_5\dot{\varphi}^5. \quad (3.69)
\end{aligned}
$$

As a side note, although it may not seem obvious from this formulation, these EFT functions don't actually depend on H_0. The H_0's in the denominator will cancel out with the H_0's which we get from the explicit expressions for the derivatives of ϕ.

Having these EFT functions allows us to now move on to the stability conditions, which rely on these functions to compute the stability of a solution.

3.4 Stability conditions

Before we give the conditions which determine the stability of a solution, it is useful to have an idea of what stability means in this context. We will be looking at two types of stabilities, namely the absence of ghosts and the absence of gradient instabilities.

Ghosts are quanta with either a negative energy or a negative norm [6]. The type of ghost we will be looking at is a negative kinetic term, which is the term in the Lagrangian with temporal derivatives. In classical mechanics, the kinetic term is given by $T = \frac{m}{2}v^2$, with m the mass of a particle and $v = \dot{x}$ the velocity. With this in mind, it's not hard to see why we want to avoid a negative kinetic energy, since this would mean that there could exist interactions which cost zero energy, thus breaking the law of conservation of energy.

In a similar way to how ghosts are terms with a wrong sign temporal derivative, gradient instabilities are terms with a wrong sign spatial derivative [6]. To give an example, solving the harmonic oscillator with a negative spring constant (the equations of motion would be $F = kx$, with $k > 0$), gives us the solutions $x = e^{\sqrt{k/m}t}$ and $x = e^{-\sqrt{k/m}t}$. Naturally, we want the solutions to the harmonic oscillator to be periodic, but the solutions with a negative spring constant are instead exponentially growing or decreasing. These are not physical solutions, so we refer to them as gradient instabilities.

For the theory of covariant Galileon Lagrangians, the stability conditions are given by:

$$\mathcal{K} > 0, \tag{3.70}$$

$$c_s^2 > 0, \tag{3.71}$$

in which

$$\mathcal{K} = \frac{A(4c(t)2A + 3(m_0^2\Omega\prime(t) + \bar{M}_1^3(t))^2 + 8M_2^4 A)}{2H(t)A + m_0^2\Omega\prime(t) + \bar{M}_1^3(t))^2}, \tag{3.72}$$

and

$$\begin{aligned} c_s^2 = \big[&4A^2\big[2c(t) + m_0^2 2H(t)\Omega\prime(t) - m_0^2\Omega''(t) + H(t)(2H(t)A + \bar{M}_1^3(t) \\ &+ \Omega''(t)4m_0^2A^2) - 2\bar{M}_3^2(t)H'(t) - 2H(t)\bar{M}_3^2\prime(t) + \bar{M}_1^3\prime(t)\big] \\ &+ 4A(2m_0^2\Omega'(t) - 2\bar{M}_3^2(t))C - 2m_0^2BC^2\big] \\ &/\big[A\big[4c(t)A + 3(m_0^2\Omega'(t) + \bar{M}_1^3(t))^2 + 8M_2^4(t)A\big]\big]. \end{aligned} \tag{3.73}$$

Here we have used some abbreviations:

$$A = m_0^2(\Omega(t) + 1) - \bar{M}_3^2(t), \tag{3.74}$$

$$B = \Omega(t) + 1, \tag{3.75}$$

$$C = 2H(t)A + m_0^2\Omega'(t) + \bar{M}_1^3(t), \tag{3.76}$$

for better readibility.
$\mathcal{K}$ is the kinetic term of our theory, thus when $\mathcal{K} < 0$, we would speak of a ghost.

The other instability, $c_s^2 < 0$, is a gradient instability which expresses that the speed of sound of the scalar field must be real.

We first want to rewrite these functions in a nicer form, which means that we want to factor out the Planck mass m_0 as much as possible, for which we will need to express these quantities in terms of our γ-functions, instead of the unscaled EFT functions. For this we introduce the scaled version of $c(t)$, which we will call $\tilde{c}(t) = \frac{c(t)}{m_0^2}$. We will similarly introduce the expressions $\tilde{A} = \frac{A}{m_0^2}$, and $\tilde{C} = \frac{C}{m_0^2}$, which gives:

$$\begin{aligned}\tilde{A} &= \Omega(t) + 1 - \frac{\bar{M}_3^2(t)}{m_0^2}\\ &= \Omega(t) + 1 - \gamma_4(t), \qquad (3.77)\\ \tilde{C} &= 2H(t)\frac{A}{m_0^2} + \Omega'(t) + \frac{\bar{M}_1^3(t)}{m_0^2}\\ &= 2H(t)\tilde{A} + \Omega'(t) + H_0\gamma_2(t). \qquad (3.78)\end{aligned}$$

Substituting these functions give us:

$$\mathcal{K} = m_0^2 \frac{\tilde{A}(\tilde{c}(t)\tilde{A} + 3(\Omega\prime(t) + \gamma_2(t)H_0)^2 + 8H_0^2\gamma_1(t)\tilde{A})}{(2H(t)\tilde{A} + \Omega\prime(t) + H_0\gamma_2(t))^2} \qquad (3.79)$$

$$\begin{aligned}c_s^2 = &\big[4\tilde{A}^2\big[2\tilde{c}(t) - 2H(t)\Omega\prime(t) - \Omega''(t) + H(t)(2H(t)\tilde{A} + H_0\gamma_2(t)\\ &+ \Omega''(t)4\tilde{A}^2) - 2\gamma_4(t)H'(t) - 2H(t)\gamma_4\prime(t) + H_0\gamma_2\prime(t)\big]\\ &+ 4\tilde{A}(2\Omega'(t) - 2\gamma_4(t))\tilde{C} - 2B\tilde{C}^2\big]\\ &/\big[\tilde{A}\big[4\tilde{c}(t)\tilde{A} + 3(\Omega'(t) + H_0\gamma_2(t))^2 + 8H_0^2\gamma_1(t)\tilde{A}\big]\big]. \qquad (3.80)\end{aligned}$$

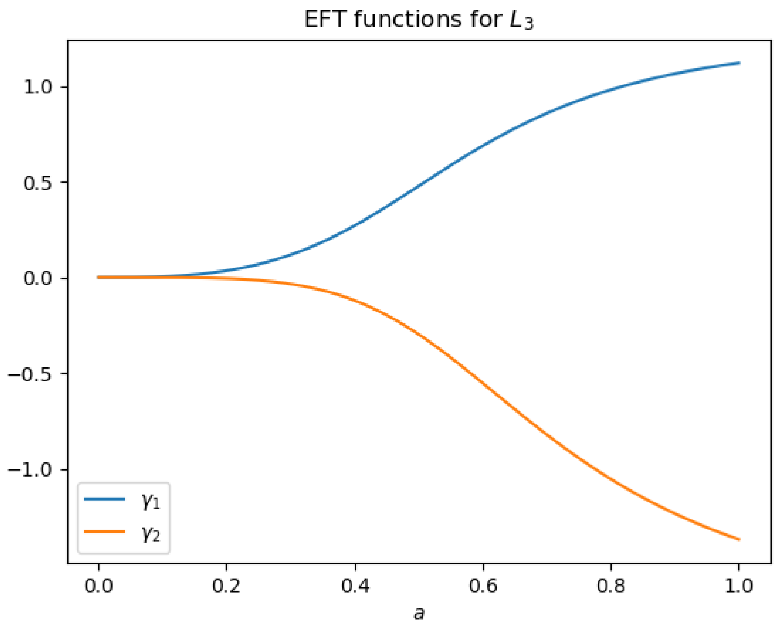
Chapter 4

Results

4.1 The EFT functions

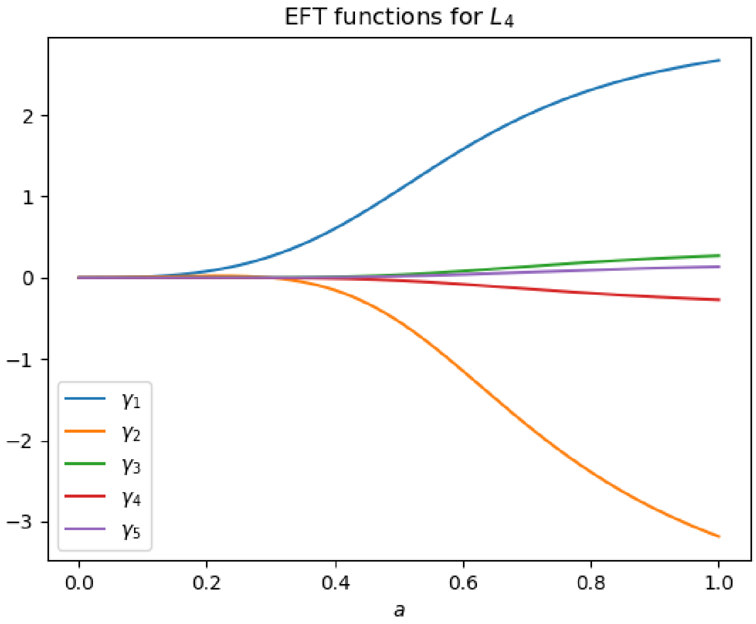
For the first part of the results we have plotted the EFT functions γ_1 to γ_5 with test values for the cosmic parameter Ω_m and the free parameters of the covariant Galileon model. We have set $\Omega_{m0} = 0.315$ [7], from which we can calculate $\Omega_{\phi 0} = 1 - \Omega_{m0} = 0.685$. Remember that L_3 is only dependent on the value of Ω_{m0}, so choosing it already allows us to plot the L_3 EFT functions, as in figure 4.1.

Figure 4.1: The non-zero EFT functions for L_3 as given in equation 3.36 and equation 3.37. The x-axis is in terms of the scale factor $a(t)$, the y-axis is dimensionless, and we have chosen $\Omega_{m0} = 0.315$.



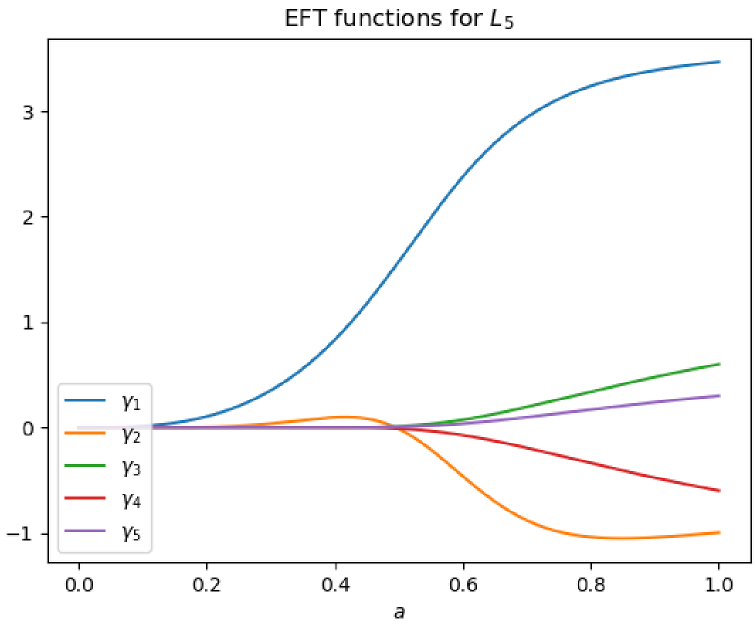
For L_4, we have an additional parameter, namely c_4. In figure 4.2, we have set it to $c_4 = 0.001$.

Figure 4.2: The non-zero EFT functions for L_4 as given in equations 3.47 to equation 3.66. The x-axis is in terms of the scale factor $a(t)$, and the y-axis is dimensionless. The parameters we've chose are $\Omega_{m0} = 0.315$ and $c_4 = 0.001$.



For L_5, we need to choose values for c_3 and for ξ. We have chosen them to be $c_3 = 0.1$ and $\xi = 2$. This gives plot as shown in figure 4.3.

Figure 4.3: The non-zero EFT functions for L_5 as given in equations 3.63 to equation 3.67. The x-axis is in terms of the scale factor $a(t)$, and the y-axis is dimensionless. The parameters we've chosen are $\Omega_{m0} = 0.315$, $c_3 = 0.1$ and $\xi = 2$.



Next, we have varied the first two EFT functions γ_1 and γ_2 for the complete case for $\xi \in (0,5]$ to get an idea of how they are dependent on ξ. We have choses this parameter to vary since it seemed to be the one with the biggest impact. γ_1 is shown in 4.4 and γ_2 in 4.5

Figure 4.4: The behaviour of γ_1 in the complete case under variation of $\xi \in (0,5]$, with the other parameters set on $\Omega_{m0} = 0.315$ and $c_3 = 0.1$. The x-axis is in the terms of the scale factor $a(t)$ and the y-axis is dimensionless. Darker colour coincides with a higher value of ξ.

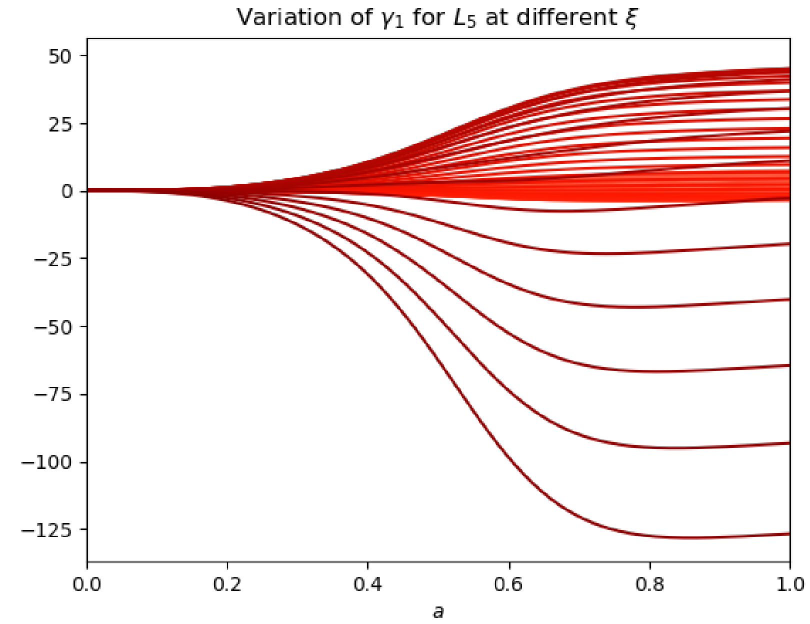
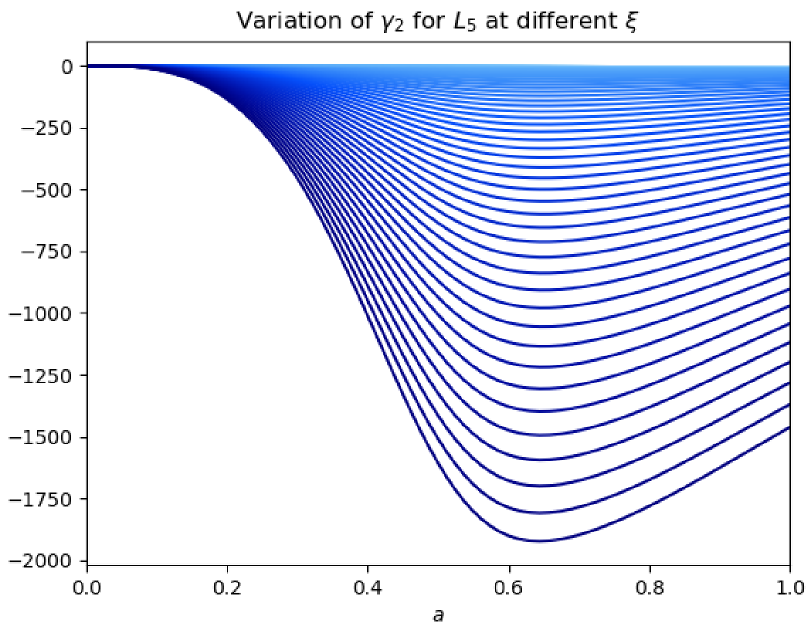


Figure 4.5: The behaviour of γ_2 in the complete case under variation of $\xi \in (0,5]$, with the other parameters set on $\Omega_{m0} = 0.315$ and $c_3 = 0.1$. The x-axis is in the terms of the scale factor $a(t)$ and the y-axis is dimensionless. Darker colour coincides with a higher value of ξ.



4.2 Stable solutions

As mentioned earlier, we calculated the sets of parameters for which the solutions are stable only for the full model, namely L_5. The free parameters of the γ functions are in this case Ω_{m0}, c_3, and ξ. In figures 4.6, 4.7, and 4.8 are the results of the condition $\mathcal{K} > 0$ at different values for the cosmic parameters Ω_{m0} the combinations of c_3 and ξ for which the solution is stable. We've chosen to vary Ω_{m0} around the experimentally obtained value of $\Omega_{m0} = 0.315$ with ± 0.17, which is ten times the interval given in [7]. The intervals for c_3 and ξ are given by $c_3 \in [0, 0.14]$ and $\xi \in [0, 5]$, which are the intervals that gave the best insight in the behaviour.

Figure 4.6: The values of c_3 and ξ for which the stability condition $\mathcal{K} > 0$ is met at $\Omega_{m0} = 0.145$.

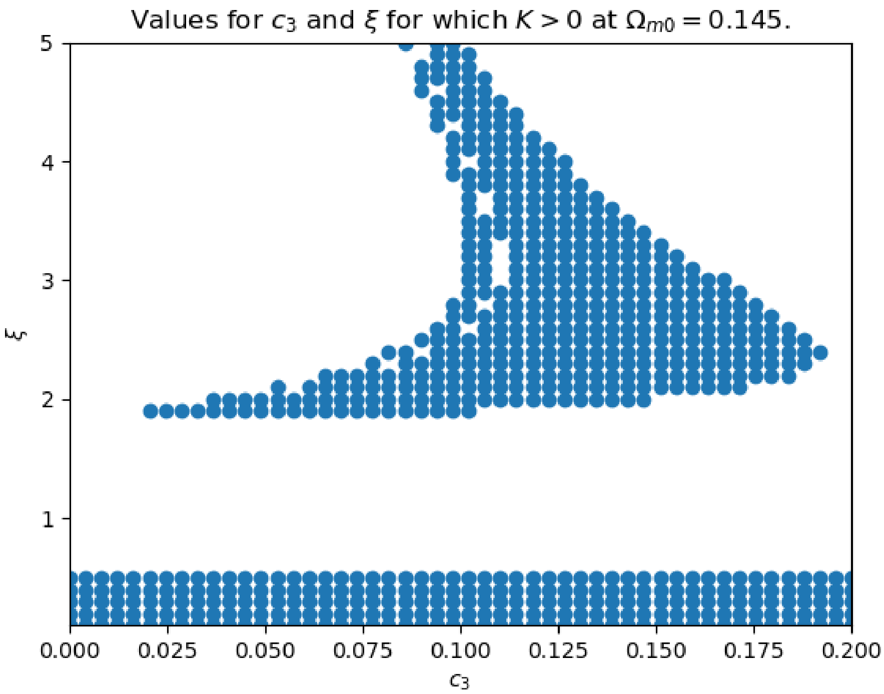


Figure 4.7: The values of c_3 and ξ for which the stability condition $\mathcal{K} > 0$ is met at $\Omega_{m0} = 0.315$.

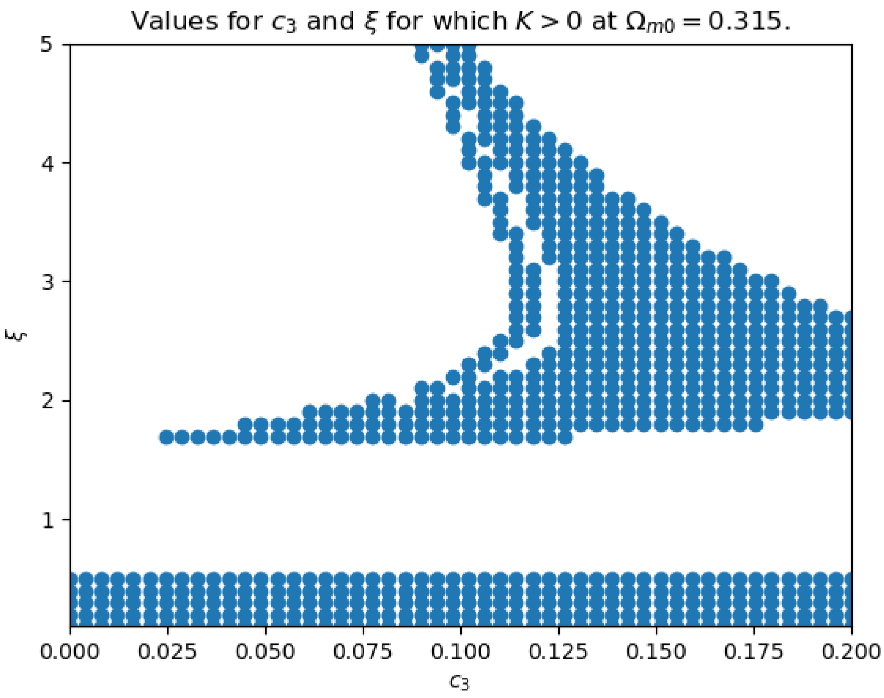
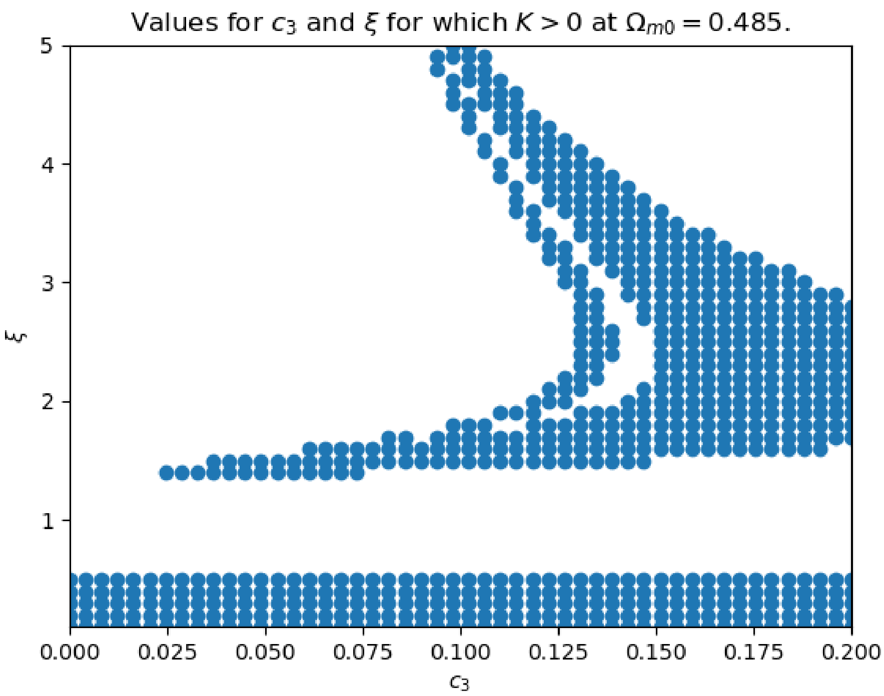


Figure 4.8: The values of c_3 and ξ for which the stability condition $\mathcal{K} > 0$ is met at $\Omega_{m0} = 0.485$.



Next, we did the same thing for the condition $c_s^2 > 0$, which can be seen in plots 4.9, 4.10 and 4.11. We have used the same parameter space as for the condition $\mathcal{K} > 0$.

Figure 4.9: The values of c_3 and ξ for which the stability condition $c_s^2 > 0$ is met at $\Omega_{m0} = 0.145$.

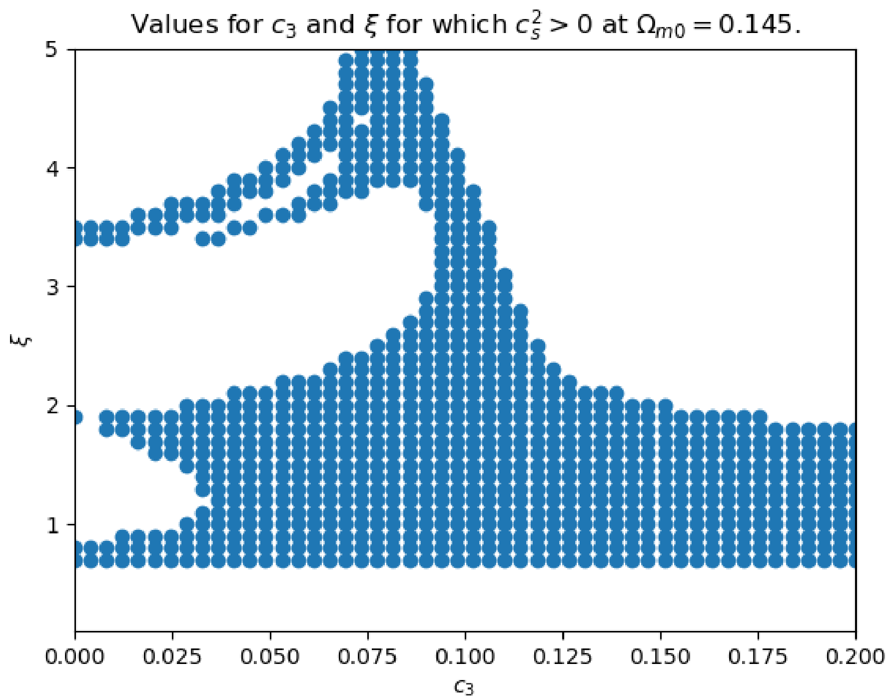


Figure 4.10: The values of c_3 and ξ for which the stability condition $c_s^2 > 0$ is met at $\Omega_{m0} = 0.315$.

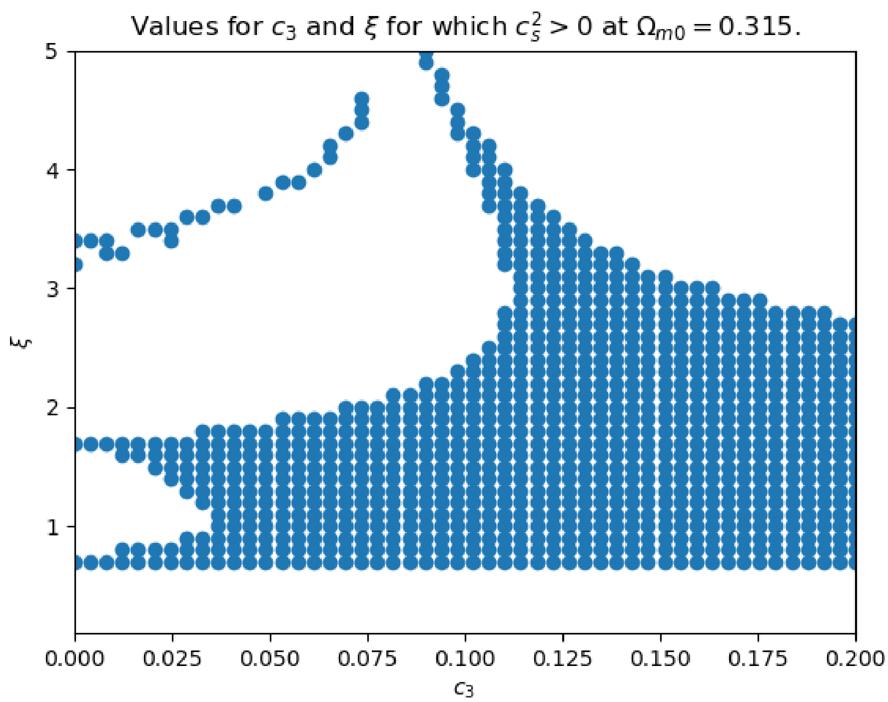
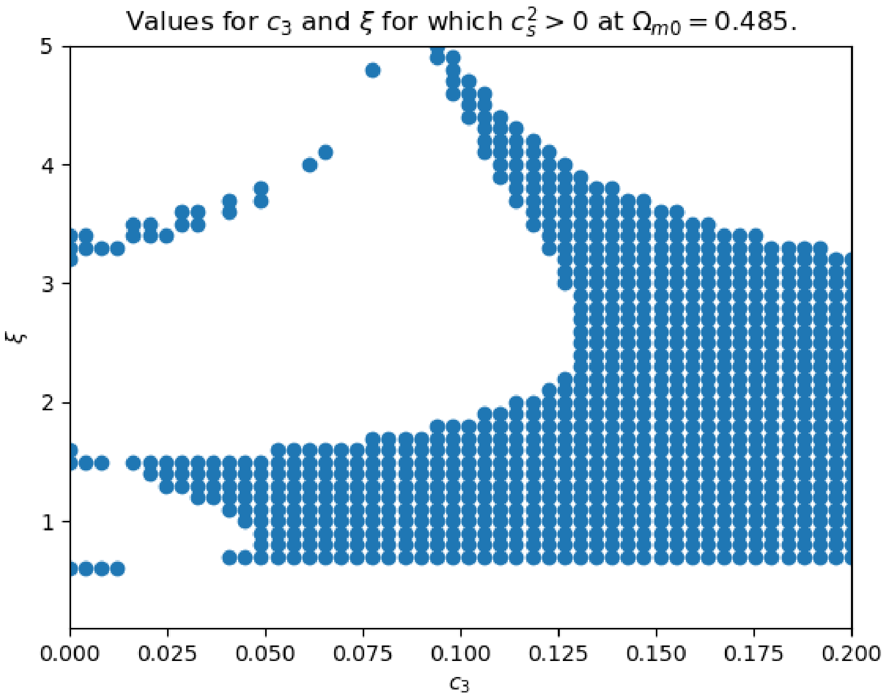


Figure 4.11: The values of c_3 and ξ for which the stability condition $c_s^2 > 0$ is met at $\Omega_{m0} = 0.485$.



Chapter 5

Discussion

The things which stand out in the first part of the results are the strange shape of γ_2 in the L_5 case in figure 4.3 and how different values of ξ have γ_1 change signs, also in the L_5 case, as seen in figure 4.4. While it seems like γ_2 scales with ξ, and is heavily dependent on it as well, as seen figure 4.5, γ_2 does not only change magnitudes quite strongly but also has a sign change. Strangely, the it starts negative (as seen in bright red), then it moves to be positive, where it reaches a maximum before becoming negative again. It might interesting to see if there is some oscillating behaviour for even larger values of ξ.

Next we look the second part of the results. By comparing the values for c_3 and ξ for which the stability conditions are met with the values for ξ and c_3 found by [1], we can draw several conclusion. The values given in Table II of [1] in the Base Quintic case are $\xi = 4.3^{+0.52}_{-1.58}$ and $c_3 = 0.132^{+0.019}_{0.004}$. In figures 4.7 and 4.9, which are at the most likely value of Ω_{m0}, these points are not included, but with their error bars they are. It is also worth noting that the lower error bar of ξ is much larger than the upper one, which coincides with our plots given more stable points for lower values of ξ.

Furthermore, it seems that the condition $\mathcal{K}$ gives a series of points at the bottom of the graphs for which the value of ξ gives a stable configuration for any value of c_3. It is at this point not clear whether this is a curiosity of the functions (note that for $\xi = 0$ we have singularity for c_5, so for ξ approaching zero c_5 will get arbitrarily large), or if it also has a physical explanation. However, since the other condition, $c_s^2 > 0$, does not give any stable points in this area, it's safe to assume we should only take the larger values of ξ into account.

Further research would include not only evaluating the stability of the solutions, but also going into how well the solutions agree with observations. The main goal is to explain the accelerated expansion of the universe, and this research hasn't answered that question yet.

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