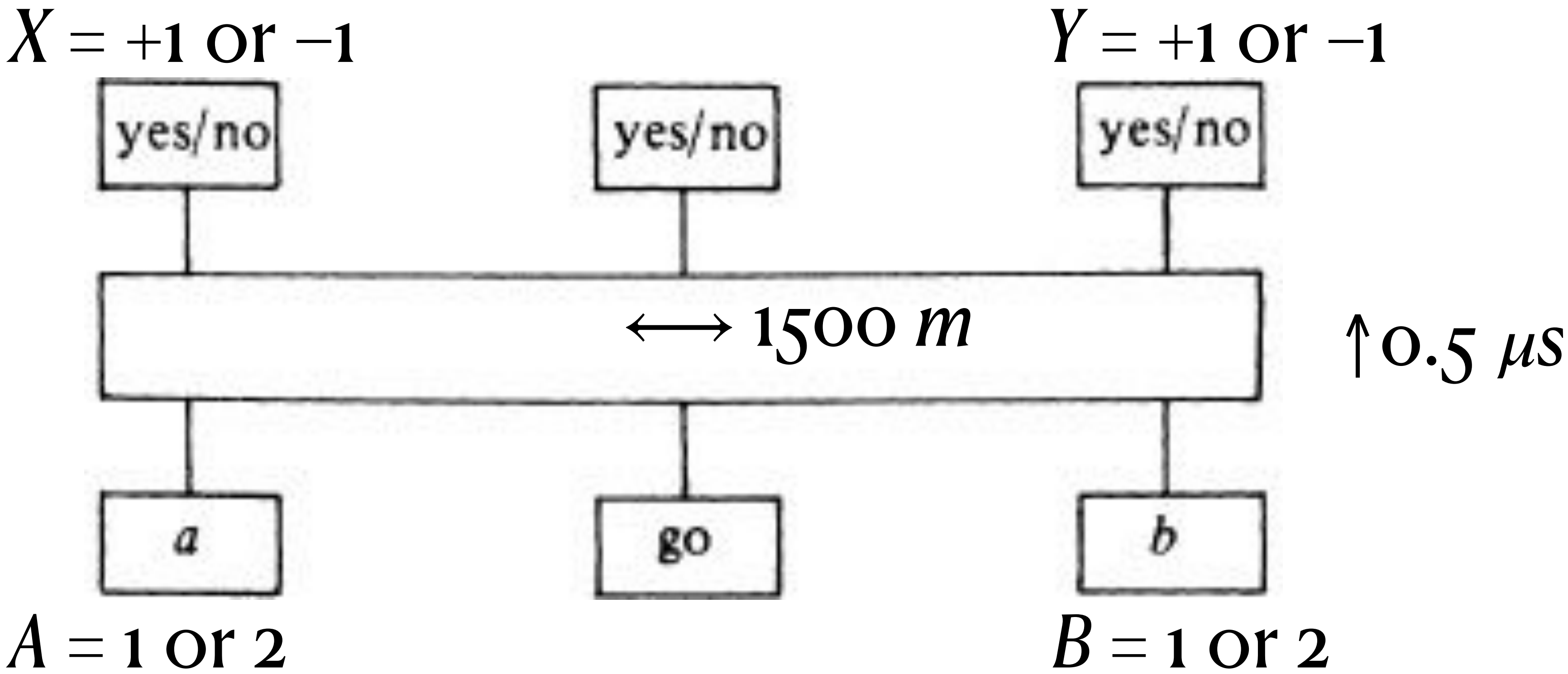
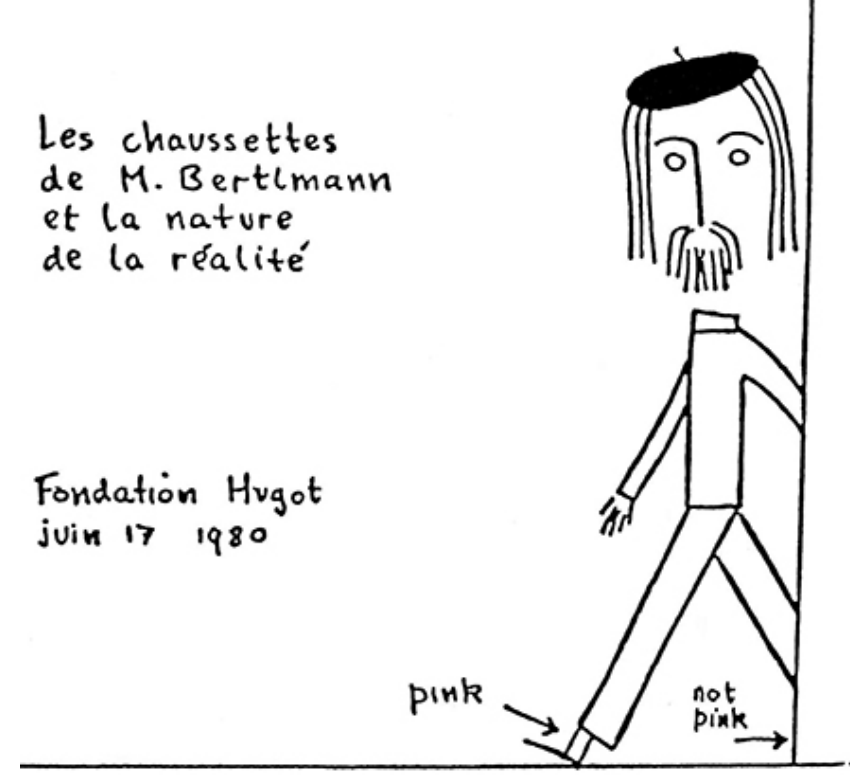
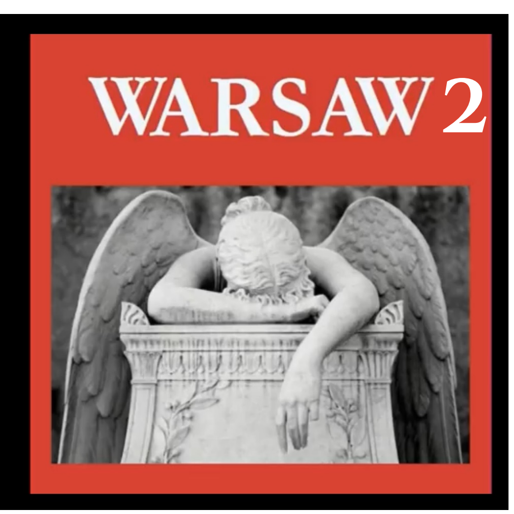
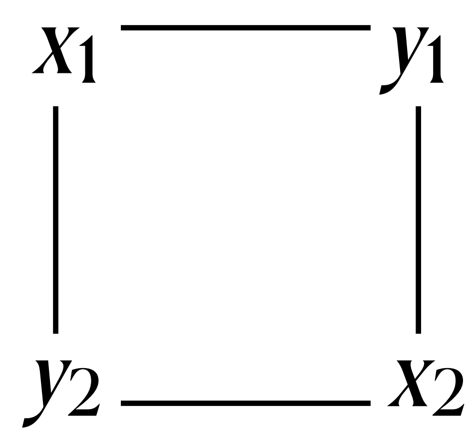






Warsaw 2

Last week's talk, continued



https://www.math.leidenuniv.nl/~gill/Warsaw2.pdf

Richard Gill, **Tuesday** 14 April 2020, 17:00 Warsaw time, Zoom from J U Krakow

[4 slides appendix to: https://www.math.leidenuniv.nl/~gill/Warsaw.pdf]

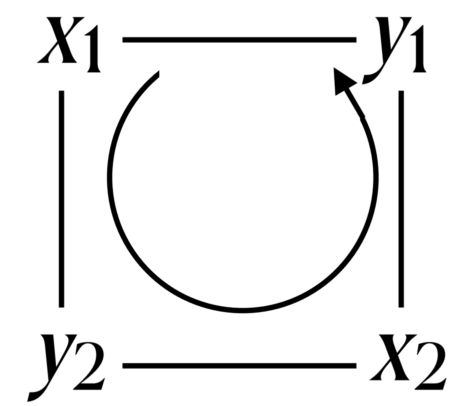
Notation

Counterfactual outcomes, factual outcomes, factual settings
Realism, locality, freedom

- A, B, X, Y are 4 binary random variables: 2 settings, 2 outcomes
- A, B take values in $\{1, 2\}$; X, Y take values in $\{-1, +1\}$
- A , B are independent fair coin tosses
- X_1, X_2, Y_1, Y_2 are Alice and Bob's **counterfactual outcomes** in the counterfactual situation that each inputs either setting
- They exist by the assumptions of **realism** and **locality**
- $X = X_A$, $Y = Y_B$
- (X_1, X_2, Y_1, Y_2) is statistically independent of (A, B) by the assumption of **freedom**

Logic & arithmetic

If three sides link equal values, so does the fourth



$$x_1 = y_2 \;\&\; y_2 = x_2 \;\&\; x_2 = y_1 \;\Rightarrow\; x_1 = y_1$$

$$\Rightarrow \quad x_1 \neq y_1 \;\Rightarrow\; x_1 \neq y_2 \text{ or } y_2 \neq x_2 \text{ or } x_2 \neq y_1$$

$$\Rightarrow \quad \mathrm{I}(x_1 \neq y_1) \;\leq\; \mathrm{I}(x_1 \neq y_2) + \mathrm{I}(y_2 \neq x_2) + \mathrm{I}(x_2 \neq y_1)$$

$$\Rightarrow \quad 2 + 1 - 2\,\mathrm{I}(x_1 \neq y_1) \;\geq\; 1 - 2\,\mathrm{I}(x_1 \neq y_2) + 1 - 2\,\mathrm{I}(y_2 \neq x_2) + 1 - 2\,\mathrm{I}(x_2 \neq y_1)$$

$$\Rightarrow \quad 2 + x_1 y_1 \;\geq\; x_1 y_2 + y_2 x_2 + x_2 y_1$$

$$\Rightarrow \quad x_1 y_2 + y_2 x_2 + x_2 y_1 - x_1 y_1 \;\leq\; 2$$

$$\mathrm{I}(x_1 \neq y_1) \;\leq\; \mathrm{I}(x_1 \neq y_2) + \mathrm{I}(y_2 \neq x_2) + \mathrm{I}(x_2 \neq y_1)$$

$$\Rightarrow \quad \mathrm{I}(x_1 \neq y_1) \;\leq\; 3 - \mathrm{I}(x_1 = y_2) - \mathrm{I}(y_2 = x_2) - \mathrm{I}(x_2 = y_1)$$

$$\Rightarrow \quad \mathrm{I}(x_1 = y_2) + \mathrm{I}(y_2 = x_2) + \mathrm{I}(x_2 = y_1) + \mathrm{I}(x_1 \neq y_1) \;\leq\; 3$$

Logic/arithmetic → Probability, expectations

From counterfactuals to factuals and to CHSH inequalities

$x_1y_2 \;+\; y_2x_2 \;+\; x_2y_1 \;-\; x_1y_1 \quad \leq \quad 2$

$\Rightarrow \quad \mathrm{E}(X_1Y_2) \;+\; \mathrm{E}(Y_2X_2) \;+\; \mathrm{E}(X_2Y_1) \;-\; \mathrm{E}(X_1Y_1) \quad \leq \quad 2$

$\Rightarrow \quad \mathrm{E}(XY \mid AB = 12) + \mathrm{E}(XY \mid AB = 22) + \mathrm{E}(XY \mid AB = 21) - \mathrm{E}(XY \mid AB = 21) \quad \leq \quad 2$

$\mathrm{I}(x_1 \;=\; y_2) \;+\; \mathrm{I}(y_2 \;=\; x_2) \;+\; \mathrm{I}(x_2 \;=\; y_1) \;+\; \mathrm{I}(x_1 \;\neq\; y_1) \quad \leq \quad 3$

$\Rightarrow \quad \tfrac{1}{4}\,\mathrm{I}(x_1 \;=\; y_2) \;+\; \tfrac{1}{4}\,\mathrm{I}(y_2 \;=\; x_2) \;+\; \tfrac{1}{4}\,\mathrm{I}(x_2 \;=\; y_1) \;+\; \tfrac{1}{4}\,\mathrm{I}(x_1 \;\neq\; y_1) \quad \leq \quad \tfrac{3}{4}$

$\Rightarrow \quad \mathrm{P}\Big(\; [\mathrm{X{=}Y}\;\&\;\mathrm{AB} \neq 11] \;\text{ or }\; [\mathrm{X} \neq \mathrm{Y}\;\&\;\mathrm{AB} = 11] \;\Big|\; (X_1, X_2, Y_1, Y_2) = (x_1, x_2, y_1, y_2) \Big) \quad \leq \quad \tfrac{3}{4}$

Repeat N times, each time conditioning on past … the total number of "successes" is stochastically smaller than a Binomial (N, ¾) random variable.

QM allows Binomial (N, q) with $q = (2 + \sqrt{2}\,)/4 \;\approx\; 0.85$