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Comment on “Quantum correlations are weaved by the spinors of the Euclidean primitives”

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I point out some obviously fatal mathematical errors in the recent paper published in this journal “Quantum correlations are weaved by the spinors of the Euclidean primitives” by Joy Christian, director of the *Einstein Center for Local Realistic Physics*, located in the city of Oxford, in the UK.

1. The Hurwitz theorem, the core of Bell's theorem, and the Bell-core versions of Gull and of Gill

I will first summarise two well known and purely mathematical results, which contradict claims in Christian (2018) [3]. I will add to those two, another two more recent and less well known results. The two well established mathematical results are: (a), the so-called *Hurwitz theorem* (so-called because it was conjectured by Hurwitz; it was only proved decades later) showing that there are only four normed division algebras \mathbb{R} , \mathbb{C} , \mathbb{H} and \mathbb{O} , see Baez (2002) [1]; (b) the mathematical core of *Bell's theorem*, Bell (1964) [2] on the incompatibility of quantum theory with local realism. The less well known results are (c) *Gull's theorem*, Gull (2016) [7], a computer science version of Bell's core result using well-known Fourier theory, and (d) *Gill's theorem*, Gill (2003) [5], an older and stronger version of Gull's theorem using well-known results from martingale theory.

There really can be no serious doubt that the purely mathematical results (a) and (b) are true, just as there is absolutely no doubt that Pythagoras' theorem is true, and absolutely no doubt that Euclid's theorem that there are infinitely many prime numbers is true. Of course, one can always dispute their applicability in some application domain, or start a philosophical discussion about what mathematical existence really means, or argue that there is some hidden defect in the presently established foundations of mathematics. But Dr Joy Christian (JC in the sequel) claims in his RSOS paper Christian (2018) that *the accepted mathematical theorems* (a) and (b) are false, and claims to provide *purely mathematical counterexamples* against them.

JC also presents a computer simulation which, if valid, contradicts (c) and (d). The two refinements of (the mathematical core of) Bell's theorem, namely Gull's and Gill's, seem to this writer to be equally secure. The last one, Gill's, has been further strengthened and refined and used by well known physicists and cited in papers published in *Nature* and other prestigious journals. But anyway, JC's mathematical argument against Bell's theorem (b) depends on the validity of his mathematical claim regarding the Hurwitz theorem (a), so if that first claim is overthrown, his whole paper vanishes in a puff of smoke. Superfluous to say, his own computer simulation is not a simulation of his own model, as it obviously cannot be, in view of Bell's theorem and the computer science translations thereof due to Gull and Gill. So JC's whole paper is "much ado about nothing", and a tangled web of nonsense.

How could such a paper have been published under the auspices of the Royal Society (the oldest still active scientific society in the world)? Were JC's claims correct, mathematics and science would be shaken to their very foundations. If his computer simulation were correct, everybody in the world with access to internet and a decent PC could have verified that JC was right - quantum correlations do have a completely classical, local, explanation. All quantum computing hype would be destroyed. All physics textbooks would have to be rewritten. No establishment conspiracy could have stopped the news from getting out.

Editors of the serious journals to which JC submits such papers should immediately realise that they are either dealing with an almost inconceivably revolutionary genius, or with a (pardon the expression) ordinary crackpot. Extraordinary claims require extraordinarily strong evidence. Does RSOS want to encourage serious debate on foundational issues in science, or does it just want to get internet clicks by publishing outrageous claims, despite the down-side that it thereby clouds serious debate by publishing superfluous noise? *O tempora, o mores!*

JC has in fact already published a paper contradicting the Hurwitz theorem in a respected pure mathematical journal *Communications in Algebra*, and that paper has already been retracted after scrutiny by some of the best experts in the field of Clifford algebras and Geometric Algebra (Anthony Lasenby, John Baez, to mention just two names). That paper is just a reworking of a section from the one I am discussing here! But there is no need to appeal to the authority of Lasenby or Baez, since in mathematics we write out all logical steps and a proof is a proof. Occasionally a mistake is made, and when found it is corrected. Mistakes have been found in all of JC's works since 2007 when he first started his mission to refute Bell's theorem. They have been exposed in peer-reviewed and published papers, and discussed and debated in various internet fora. JC's style is to never acknowledge a mistake, but always to respond with a "rebuttal", which tends to be longer and more unintelligible, though sporting impressive technical language. Moreover, he tends never to respond to the mathematical details spelled out by one of his critics, or to the mathematical details in the existing proofs of the just mentioned theorems. In particular, he has not shown us where modern proofs of the "Hurwitz theorem" go wrong.

Till recently, almost none of his *arXiv* preprints got published. Just recently, however, there has been a spate of publications. What has happened? The content of those papers is longer and more obscure and more ambitious than ever before, the errors are more numerous. I am only aware of one notable established academic who seems to think that there is something in them, but I am pretty sure that this is wishful thinking on their part. I think there is a problem in modern academic publishing and peer review. The rise of the predatory journals has led to serious academic publishing institutions fighting back under the guise of open access and open

science. By actually publishing referee reports (and sometimes referee names), and publishing author's responses to those reports, they are effectively absolving the scientific editors of scientific responsibility. Well, that can be done, but that will degrade the status of such journals to the level of expensive vanity publishing. Wealthy authors pay for the privilege of an expensive label. But the most expensive wine is not necessarily the best wine. A Dutch saying is "the more expensive, the more sour" (*hoe zuurder, hoe duurder*).

(a) The Hurwitz theorem

The best reference for this theorem is John Baez' wonderful paper [1] on the octonions, which starts with excellent expository material. According to Baez's Theorem 1, the real numbers, the complex numbers, the quaternions, and the octonions are the only normed division algebras (up to isomorphism, of course). Baez' definition of a normed division algebra is a real vector space endowed with a compatible multiplication operation and a norm, making it a normed vector space, and such that the norm is multiplicative; the algebra need not be commutative or even associative. The useful thing to know here is that a division algebra has no zero divisors. There do not exist elements A and B , neither equal to zero, such that $AB=0$. JC's algebra has an element called the "pseudo-scalar", I will denote it by M , such that $M^2=1$. It follows that $0=M^2-1=(M-1)(M+1)$. Taking norms, $0=\|M-1\|\cdot\|M+1\|$. Hence $\|M-1\|=0$ or $\|M+1\|=0$. Therefore $M-1=0$ or $M+1=0$, which implies that $M=1$ or $M=-1$. That is a contradiction. JC's algebra is associative, the octonions are not. JC's algebra is $Cl_{(0,3)}(\mathbb{R})$, the well studied even sub-algebra of $Cl_{(4,0)}(\mathbb{R})$. In fact, his \mathcal{K}^+ and \mathcal{K}^- are the *same* algebra; he just defines them starting from a different choice of vector space basis. The linear spans of those two bases are trivially the same. The multiplication operation is the same. This obvious fact should already set warning lights to almost anyone in a STEM discipline. Is no editor of RSOS aware of the definition of a vector space?

JC would say "the devil is in the details". He will certainly come up with an impressive "spiel" in response. I can understand that any non-mathematician would throw up their hands in confusion, and many would be silenced by the barrage of abstract mathematician's terminology. There are indeed some subtleties involved here, but anyone who understands the standard definition of "normed vector space" should be able to follow Baez's very careful and complete introduction to this theorem, of which he moreover gives the complete proof.

(b) Bell's theorem: the mathematical core

Bell (1964) [2], as rapidly improved by Clauser, Horne, Shimony and Holt (1969) [4], essentially proves the following theorem. Suppose that X_a and Y_b are a family of random variables on a single probability space, taking values in the set $\{-1, +1\}$, and where a and b denote directions in ordinary 3D Euclidean space, represented by unit vectors a, b . Then it is not possible that $\mathbb{E}(X_a Y_b) = -a \cdot b$ for all a and b .

Bell's proof works by focussing on two choices for a and two for b , delivering us four combinations of possible values of the pair (a, b) . Since four binary random variables are supported by a discrete probability space with just 16 elementary outcomes, a proof (using for instance the so-called CHSH inequality) can be framed in absolutely elementary terms. No calculus is needed. No summation of infinite series. No knowledge of physics and in particular, no knowledge of quantum physics. (On a technical note: Bell's (1964) three correlation inequality is a corollary of the later four correlation inequality known as the CHSH-Bell inequality, which was later fully espoused by Bell himself.)

Bell used his mathematical result to show that quantum mechanics violated the meta-physical principles of locality and realism. Physical implications, however, are not important in this brief note.

(c) Gull's theorem

Consider a computer simulation of a Bell-CHSH type experiment. Initially, one could imagine three computers, a source computer sending information to two measurement locations, where two computers each simulate an apparatus which receives “stuff” from a source, and a “setting” (a measurement direction) supplied by an experimenter. The “measurement station computers” each output an “outcome” ± 1 . After a large number of trials, one collects the inputs (settings) and outputs (“outcomes”) together and computes the correlation (meaning in this context, just the mean value of the product) between the outcomes, for each possible pair of input settings.

Now, if that could be done, one could also create a copy of the source measurement station, including all the data which is stored on it at the beginning of the computer simulation, and then merge the two identical copies of the source, with the two measurement stations, giving us together two completely separated classical computers, which perform the following task.

The two computers are both loaded with data and a computer program, and then disconnected. After that, in N rounds, each computer is supplied an input, and each computer supplies an output. After the complete run of N “trials” is completed, one collects all the data together, and correlates the outputs, for each possible pair of inputs. Gull [7] used to pose the question, as part of the “Part III” exam in Theoretical Physics at Cambridge University: is it possible to recover, in the limit, the correlations $-a \cdot b$? He outlines the proof that this is impossible, using a really nice argument from Fourier analysis. Unfortunately he never bothered to publish a formal proof, we must make do with his overhead transparencies from a conference, and a further worked out proof on [stackexchange](#) [8]. Inspection of Gull's outline proof shows that he is pretty explicitly making a particular “no use of memory” assumption. Each of those computer's outputs, in trial n , depend only on the initial data stored in the computers and their programs, and on the new setting a or b , and on the trial number n , but not on the previous $n - 1$ inputs. Here are two student exercises:

Exercise 1: write out a rigorous proof and have it published in a peer reviewed journal.

Exercise 2: investigate JC's computer code and see why the code could not be separated into two programs running on two separated computers

Hint: Notice the line `if (lambda==1) q=(NA NB) else q=(NB NA)`

(d) Gill's theorem

Gill (2003) [5] was bothered by several eminent scientists claiming that Bell's theorem was false, and some of them claiming to even be able to prove this by simulated computer models, running on a distributed network of computers. Some of them claimed that Bell had not taken account of “time” in his theory (not true, Bell discusses that explicitly, though not perhaps in that famous first paper which made Bell famous and which, for many opponents of Bell, is the only publication by him which they bother to read). Gill wanted to make a bet with some of them that this couldn't be done, but he didn't want to lose his bet. He was therefore bothered by inevitable statistical variation, and also by the possibility that the computer programs might make use of information about the past $n - 1$ settings and outcomes. Using martingale theory he proved a probability bound showing that deviations from the CHSH inequality of any size had exponentially small probability, provided only that the binary setting choice at each trial was performed, outside of each measurement station, and again and again, by a fair coin toss. The measurement station computers were even allowed to communicate with one another *between* each trial. This result was later refined and improved and used by the experimenters in the four famous “loophole free” Bell experiments of 2015, including one published in *Nature*, which acknowledged my contribution and cited my original paper of 15 years earlier.

2. Conclusion

An internal investigation should be held to find out how this disaster could have happened. On a constructive note, I suggest that some *RSOS* editors (as well as editors of *IEEE Access*) need to be recruited from mathematics, and from quantum information/quantum computation/foundations of quantum mechanics. The community of serious scientists working on Bell's theorem need to think seriously about how to improve public outreach; science journalists too need education and need good resources. Too much meaningless quantum hype is produced by overzealous university PR departments.

Ethics. In a paper about scientific ethics, it is inevitable that some scientists get criticised. The author sees his task as necessary, even if unpleasant, and accepts questioning of his own motives and qualifications.

Data Accessibility. This article has no additional data.

Authors' Contributions. Not relevant.

Competing Interests. The present author RDG has submitted an accusation of plagiarism concerning other work of JC, published in *IEEE Access*. This issue has been debated on several public internet fora. JC denies the accusation.

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Disclaimer. RDG is angry with himself that he did not accept a request to referee this paper when it was first submitted to *RSOS*. He felt then that it was time that other people read, and reacted to, JC's papers, since it wasn't healthy that he had become the "go-to" referee for editors looking for justification to reject yet another flawed attempt by JC to disprove Bell's theorem. RDG had moreover already placed criticism of JC's *arXiv* preprint on *PubPeer*, and later published a paper Gill (2020) [6] containing an analysis of a whole series of papers by JC. The *arXiv* version of that paper includes a correction note.

References

1. J.C. Baez (2002). *Bull. Amer. Math. Soc.* 39 (2002), 145-205
2. J.S. Bell (1964). On the Einstein Podolsky Rosen paradox, *Physics* 1 (3), 195–200.
3. J. Christian (2018). Quantum correlations are weaved by the spinors of the Euclidean primitives, *R. Soc. Open Sci.* 5, 180526 (40 pp.)
<https://royalsocietypublishing.org/doi/full/10.1098/rsos.180526>
4. J.F. Clauser, M.A. Horne, A. Shimony, and R.A. Holt (1969). Proposed experiment to test local hidden-variable theories, *Phys. Rev. Lett.* 23, 880–884.
<https://doi.org/10.1103/PhysRevLett.23.880>
5. R.D. Gill, (2003). Accardi contra Bell: the impossible coupling. pp. 133-154 in: In M. Moore, C. Leger, and S. Froda (Eds.), *Mathematical Statistics and Applications: Festschrift for Constance van Eeden*, Lecture Notes–Monograph series, Hayward, Ca. Institute of Mathematical Statistics. <https://arxiv.org/abs/quant-ph/0110137>
6. R.D. Gill (2020). Does Geometric Algebra provide a loophole to Bell’s Theorem? (with correction note). <https://arxiv.org/abs/1203.1504> *Entropy* 2020, 22(1), 61 (21 pp.);
<https://doi.org/10.3390/e22010061>
7. S. Gull (2016). Quantum acausality and Bell’s theorem. Four pages pdf of overhead slides. <http://www.mrao.cam.ac.uk/~steve/maxent2009/images/bell.pdf>. From personal webpage with several items of work from past conferences.
<http://www.mrao.cam.ac.uk/~steve/maxent2009/>
8. Question posted on “Stackexchange” by “Eran Medan” (2020), reply by “Chiral Anomaly” (2020) <https://physics.stackexchange.com/questions/547039/help-understanding-prof-steve-gulls-explanation-of-bells-theorem>