Lucia: Killed by Innocent Heterogeneity

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Abstract

I investigate the consequences of allowing innocent heterogeneity between nurses. A modest amount of variation makes the chance that an innocent nurse experiences as many incidents as the number Lucia actually did experience, the somewhat unremarkable *One in Nine*.

Introduction

I suppose that nurses have a Gamma distributed incident-risk per shift: the Gamma distribution, bland and unremarkable, is chosen for mathematical convenience. For these very reasons this is the default assumption when studying overdispersion in applied statistics. I assume that the number of incidents experienced by a given nurse in many shifts is Poisson distributed, an uncontroversial assumption since incidents are rare and probably unrelated from one patient to another. I'll choose the mean risk-per-shift to match the overall (pooled) average rate of incidents, and I'll take the Gamma distribution to have a coefficient of variation equal to 1, which roughly speaking means that it is easy that one nurse has twice the incident rate of another. I have almost no objective data supporting this number. Most nurses and nursing specialists I have talked to, believe that innocent heterogeneity in actually experienced incident rates could be large, especially over moderately short periods of time (a couple of years, say). On the other hand, a senior medical specialist, expert for the prosecution, stated categorically that well trained nurses are interchangeable. (Sir Roy Meadow was also a respected medical specialist, expounding on probability.) There is no contradiction between these opinions if one takes account of the fact that nurses have a

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large amount of influence in determining their shifts, that severity of case-mix might well fluctuate over time together with the amount of work-time put in by some nurses, and so on and so forth.

I use Derksen and de Noo's revised data set, taking account of incidents among the other nurses (which formerly were taken to be, by definition, *not suspicious*), and removing incidents and deaths for which Lucia was deemed innocent (not charged with murder or attempted murder, presumably because these events were medically speaking "expected to happen, when they actually did").

This may lead to some small bias in favour of Lucia in my analysis. The pooled incident-rate would be an overestimate if Lucia is guilty — however, there is external evidence that it is quite normal. Derksen and de Noo's definition of "incident" is mildly biased in favour of Lucia since incidents which occurred during other nurses' shifts wouldn't *necessarily* be counted as attempted murders if the court and the medical experts and the police had known for sure that Lucia was on duty at that time.

On the other hand, I do not do any post-hoc correction, though a correction is still called for, since *we are only computing this probability, because we noticed that something unlikely appears to have happened.* My estimate of the heterogeneity is "sucked out of my thumb", as the Dutch say; however, pooling three wards at two different hospitals certainly causes natural heterogeneity between incident rates of nurses (nurses in different wards, different kinds of hospitals even, surely experience different rates). Thus, the amount I have assumed here could well be conservative. We just don't know. *If* Lucia is innocent, *then* we actually do have objective data, and it supports a large variation. I feel that my guestimate is pretty neutral.

The math

Here, I perform the standard derivation showing that a Gamma mixture of Poissons is Negative Binomial, and then I write down the familar relation between NegBin and Bin. My notation is all more or less conventional. I take the two parameters of the Gamma distribution to be the shape parameter and the inverse scale parameter, respectively. I take the parameters of the Negative Binomial to be the number of heads which we're waiting for, and the probability of heads, respectively; the random variable itself stands for the number of tails we observe before getting the prescribed number of heads (as is well known, we need not take this last parameter to be an integer, if we replace factorials with gamma functions in the formulae). The parameters of the Binomial are the number of tosses and the probability of heads, respectively.

Firstly, from a Gamma$(\rho, \rho/\mu)$ distributed (thus, mean μ and variance μ^2/ρ) mixture of Poisson(λ)'s, to a Negative Binomial:

$$
\begin{aligned}
\Pr(k \text{ incidents}) &= \int_{\lambda=0}^{\infty} \frac{e^{-\lambda}\lambda^k}{k!} \frac{(\rho\lambda/\mu)^{\rho-1}e^{-\rho\lambda/\mu}}{\Gamma(\rho)} \rho \, \mathrm{d}\lambda/\mu \\
&= \frac{(\rho/\mu)^\rho}{k!\Gamma(\rho)} \int_{\lambda=0}^{\infty} e^{-(1+\rho/\mu)\lambda} \lambda^{k+\rho-1} \mathrm{d}\lambda \\
&= \frac{(\rho/\mu)^\rho}{k!\Gamma(\rho)(1+\rho/\mu)^{k+\rho}} \int_{\lambda=0}^{\infty} e^{-(1+\rho/\mu)\lambda} \big((1+\rho/\mu)\lambda\big)^{k+\rho-1} (1+\rho/\mu)\mathrm{d}\lambda \\
&= \frac{(\rho/\mu)^\rho \Gamma(k+\rho)}{k!\Gamma(\rho)(1+\rho/\mu)^{k+\rho}} \\
&= \binom{k+\rho-1}{\rho-1} \left(\frac{\rho}{\mu+\rho}\right)^\rho \left(\frac{\mu}{\mu+\rho}\right)^k \\
&= \Pr\left(\mathrm{NegBin}\Big(\rho, \frac{\rho}{\mu+\rho}\Big) = k \right) .
\end{aligned}
$$

Secondly, the relation between Negative Binomial and ordinary Binomial, for the special case of integer parameter $\rho = n$:

$$
\begin{aligned}
\Pr\big(\mathrm{NegBin}(n,p) \geq k\big) &= \Pr\big(\mathrm{Bin}(n+k-1,p) \leq n-1\big) \\
\Pr\big(\mathrm{NegBin}(\rho, \, \rho/(\mu+\rho)\,) \geq k\big) &= \Pr\big(\mathrm{Bin}(\rho+k-1, \, \rho/(\mu+\rho)\,) \leq \rho-1\big) .
\end{aligned}
$$

In particular, when $n = \rho = 1$,

$$
\begin{aligned}
\Pr\big(\mathrm{NegBin}(1, \, 1/(\mu+1)\,) \geq k\big) &= \Pr\big(\mathrm{Bin}(k, 1/(\mu+1)\,) = 0\big) \\
&= ((\mu/(\mu+1))^k .
\end{aligned}
$$

The numbers

Combining the Juliana Children's Hospital and the two wards of the Red Cross Hospital, Lucia had 203 shifts, 7 incidents. It is not clear whether this combination works out pro or contra Lucia (this depends on whether she did proportionately more or less shifts at the different wards, and whether the overall mean incident rate is larger or smaller at each ward; cf. Simpson's paradox). We do need to do a combined analysis; the alternative to quick and dirty "just add everything up", is a much more complicated analysis and certainly requires making more, unsupportable, assumptions. I'll take the overall probability of an incident per shift to be the ratio of total incidents to total shifts, 23/1681. Thus an average, innocent Lucia would experience $203 * 23/1681 = 2.777513$ incidents. The $\Gamma(\rho, \rho/\mu)$ distribution has mean μ, variance μ^2/ρ. I'll take $\mu = 2.777513$, $\rho = 1$, $k = 7$. The required probability is $(\mu/(\mu+1))^k = 0.1162$. Smaller than one in eight, larger than one in nine. A rounder number, not suggesting so much precision, would be one in ten. But I think this is unfair: I'll go for *one in nine*.

Conclusion: One in Nine will Hang

A modest amount of variation makes the chance that an innnocent nurse experiences at least as many incidents as the number Lucia actually did experience, the somewhat unremarkable *one in nine*.

The fact that this amount of heterogeneity easily explains the coincidence, is strong evidence that it does truly exist: the intuition of medical specialists needs to be challenged and the experience of nurses and nursing specialists is supported.

It is clear that being different, kills you. Fortunately, to paraphrase the words of Brian: "we are all individualists" (or we will be soon, when we have thrown the rest of the unlucky nurses into jail).

References

`To be completed`. Some references: my web site for the latest numbers of Derksen and de Noo and recent statistical discussions; Meester et al. for a lot more statistical discussions and calculations on the basis of the old numbers of Elffers and the public prosecution; Aart de Vos for the Gamma distribution (it is of course the usual trick when studying possible overdispersion of count data in economics, biology, medicine, ...); Derksen's book for shredding of the medical, statistical, and psychological data, and exposure of the logical fallacies and circular reasoning in the court's motivation of the guilty verdict `to be continued`.

Appendices

Post-hoc correction

`To be completed`. Suppose we experience a random event with probability uniformly distributed between zero and one, i.e., a completely unremarkable event. However we get excited when the probability is small. Let's say that a larger number of zeroes in a small probability makes us more excited. Let's compute the probability that we get so excited that we report an extremely unlikely event, and let's compute the ratio between its actual probability and the reported probability, when we do this. To do this, I'll have to come up with a model linking probability to excitement, it has to be intuitively convincing and mathematically convenient.

Let's suppose we also get excited when the probability is also rather close to one, or rather close to exactly one half, and perhaps a few more surprising coincidences. Now suppose we observe a uniformly distributed point in a unit hypercube, and we get excited when any of a number of surprising coincidences, polynomial or exponential in the dimension, happen (after all, life is very complex – high dimensional – so unlikely combinations of events are probably happening rather often) ... `to be continued`.

Gamma incident rate, with coefficient of variation one

`To be completed`. I stated that the interpretation of my choice of parameter of the gamma distribution, is that one nurse easily experiences twice as many incidents as another, which seems to be a rather conservative assumption in view of my discussions with nursing specialists. I basicly want to know what is the probability, when we pick two random numbers from Gamma(1,1) = Exponential(1), that the larger is at least twice the smaller. An easy exercise. I wonder if it is much smaller or larger than half.

Here's the answer: $1/e = 0.3679$, which is even a bit smaller than a half, but still larger than a third. I think my assumption is not unfair to the prosecution at all. Anyway, the onus is on them to prove that there is little or no heterogeneity (you are presumed innocent till you are proved guilty, I thought) ... `to be continued`.

The total number of deaths

`To be completed`. In the three years that Lucia worked at the Juliana Children's Hospital there were 6 deaths on her ward. In the three years before, there were 7. This seems pretty strong evidence that Lucia was not a serial killer on a mega-killing-spree (unless there is a second killer still loose ...). This data was presumably known in advance to the hospital director who reported Lucia to the police, and could easily have been uncovered by the public prosecutor, but it was never revealed during her trial, even though the statisticians giving contra-expertise on behalf of the defense, did call for it. They were ignored. How to incorporate it?

The world has also been informed that Lucia worked on a medium care ward, where people were not very sick, so there should not have been any deaths at all. In fact, there are more deaths in medium care than in intensive care. This fact was presumably also known to the hospital director, the police, and the prosecution, but deemed irrelevant ... `to be continued`.

Double blind randomized trial

`To be completed`. The story of Pfizer's miracle drug and billion dollar loss ... `to be continued`.

Simpson's paradox

`To be completed`. Simpson's paradox: taking account of hidden confounders can reverse the direction of the effect ... repeatedly ... Causality from correlation? ... `to be continued`.