

# Mock exam Algebraic Number Theory 2014

Solutions to this mock exam will be discussed in the exercise class of Tuesday 16 December. You are strongly advised to make this exam individually ahead of time. During the exam the use of lecture notes is allowed. Moreover, the use of a **non-graphing** calculator will also be allowed. In case of doubt, contact us well in advance. The exam will take place on January 27 from 10:00 to 13:00 in room FG1 (Medical Faculty) at the Vrije Universiteit Amsterdam.

**Exercise 1.** Give all ideals of the ring  $\mathbb{Z}[\sqrt{-10}]$  that contain the element 6.

**Exercise 2.** Determine all pairs  $(X, Y) \in \mathbb{Z}^2$  such that  $X^2 + 7 = Y^5$ .

**Exercise 3.** Let  $K = \mathbb{Q}(\sqrt[3]{28})$ , let  $\mathcal{O}_K$  be its maximal order and let  $R = \mathbb{Z}[\sqrt[3]{28}]$ .

- (a) Prove that the prime 2 is ramified in  $\mathcal{O}_K$ .
- (b) Compute the index  $[\mathcal{O}_K : R]$ .
- (c) Give an explicit finite set of generators for  $\mathcal{O}_K$  as a ring.

**Exercise 4.** Let  $L = \mathbb{Q}(\sqrt{69})$  and let  $\mathcal{O}_L$  be its maximal order.

- (a) Calculate the class group of  $L$ .
- (b) Calculate the unit group of  $\mathcal{O}_L$ , i.e., calculate its rank, give a generator of the torsion subgroup and give a set of fundamental units.