Choose three exercises from paragraph 24 and 25 in Cassels and also do the exercise below as a fourth.

F.

For $D = -19$ and $D = -47$, do the following. You are allowed to use Sage.

(1) Determine all $\tau \in \mathbb{C}$ in the standard fundamental domain for $\mathrm{SL}_2(\mathbb{Z})$ such that the lattice $L = \langle 1, \tau \rangle \subset \mathbb{C}$ has complex multiplication by $\mathcal{O}_D$.

(2) Use the lattices found to compute the Hilbert class polynomial

$$H_D = \prod_{\tau} (x - j(\tau)).$$

In order to evaluate the j-invariant $j(L) = j(\tau)$, define the function j in Sage by

```
j = lambda tau, k: j_invariant_qexp(k)(e^(2*pi*I*tau)).n()
```

This function takes as input a complex number τ and a precision k for which the expansion of

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + \dots$$

is cut off at q^k, and where we use $q = e^{2\pi i \tau}$.