GAPS IN ORBITS OF $N$-EXPANSIONS

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Abstract. For $N \in \mathbb{N}$ and $\alpha \in \mathbb{R}$ such that $0 < \alpha \leq \sqrt{N} - 1$, the function $T_\alpha : [\alpha, \alpha + 1] \to [\alpha, \alpha + 1]$ is defined as $T_\alpha(x) := \frac{N}{x} - d(x)$, where $d: [\alpha, \alpha + 1] \to \mathbb{N}$ is defined by $d(x) := \lfloor \frac{N}{x} - \alpha \rfloor$.

For $N \leq 7$ there are intervals $(a, b) \subset [\alpha, \alpha + 1]$ and $n_0 \in \mathbb{N}$ such that $T_n \cap (a, b) = \emptyset$ for all $n \geq n_0$. These gaps $(a, b)$ are investigated in the unit square $\Upsilon_\alpha := [\alpha, \alpha + 1] \times [\alpha, \alpha + 1]$, in which the elements of the orbits $T_k(x), k = 0, 1, 2, \ldots$ of numbers $x \in [\alpha, \alpha + 1]$ are connected by line segments, so as to form a web graph. The square $\Upsilon_\alpha$ is divided into cylinders $\Delta_d$, according to the value of $d$ in the function $T_\alpha$. In the case of two cylinders there may be one or no gap on $[\alpha, \alpha + 1]$; in the case of three cylinders, there are either none, one, two or three gaps, depending both on $N$ and $\alpha$; in the case of four cylinders there are usually no gaps, but when they occur, they are extremely wide; in the case of five or more cylinders no gaps occur.

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