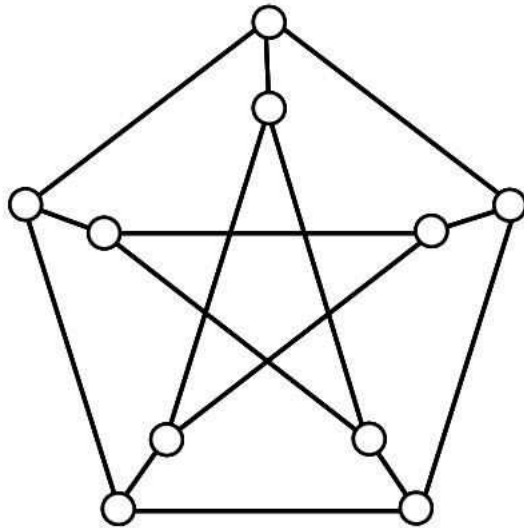
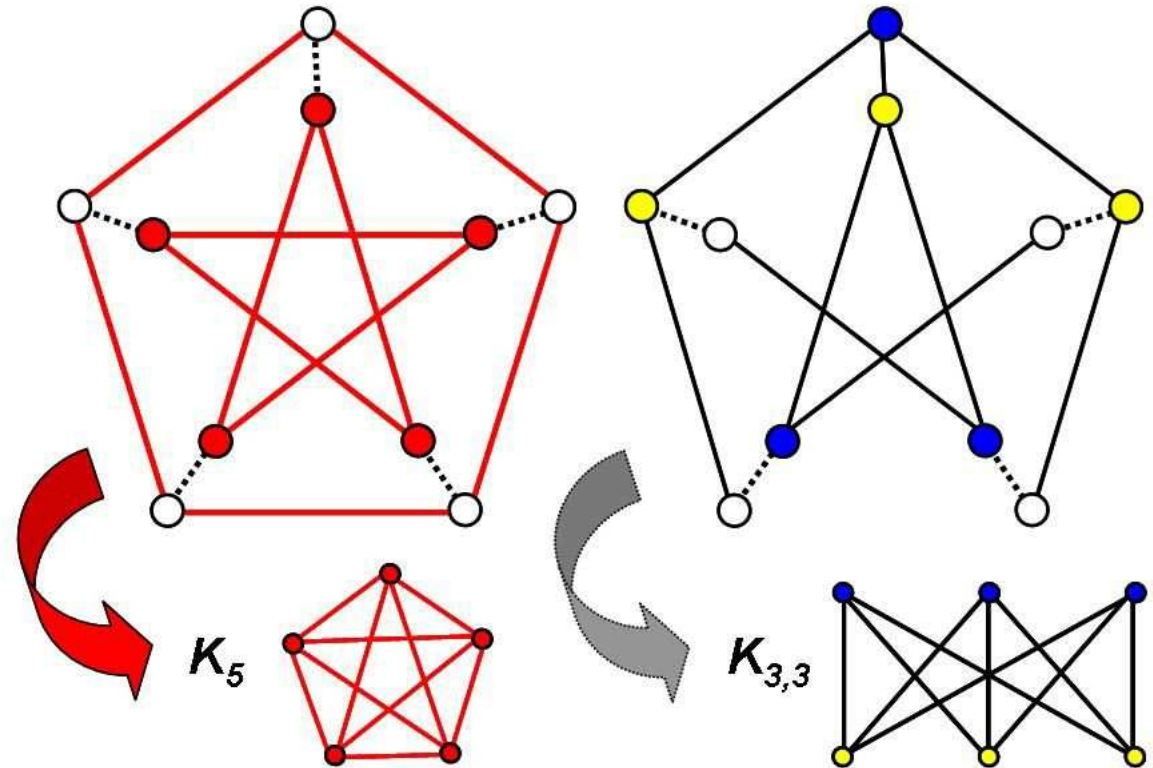


# THEOREM OF THE DAY

**Wagner's Theorem** *A graph  $G$  is planar if and only if it contains neither  $K_5$  nor  $K_{3,3}$  as a graph minor.*



The Petersen Graph



Is the graph on the left *planar*? Wagner showed that Kuratowski's theorem could be applied but with topological minors replaced by *graph minors*.  $H$  is a graph minor of  $G$  if  $G$  has a subgraph which can be reduced to  $H$  by *contracting* edges (i.e. shrinking them until their end-vertices coincide). In the picture, both  $K_5$  and  $K_{3,3}$  can be seen, by contracting the dotted edges, to be graph minors of the famous Petersen graph, but only  $K_{3,3}$  is a topological minor.

Seven years after Kuratowski proved his famous characterisation of planar graphs in terms of forbidden topological minors, Klaus Wagner proved that graph minors could also characterise planarity. The difference may appear subtle but this led Wagner to make the enormously bold conjecture, which is false for topological minors, that for any property characterised by graph minors the set of minors could be taken to be *finite*. A proof by Robertson and Seymour was completed in 2004 with the publication of the last in a series of twenty papers running to over 500 pages and spanning almost 20 years.

**Web link:** [www.ams.org/featurecolumn/archive/gminor.html](http://www.ams.org/featurecolumn/archive/gminor.html)

**Further reading:** *Introduction to Graph Theory, 4th Ed.*, by Robin Wilson, Longman, 1996.