Exercise

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In this exercise you are asked to give detailed proofs of a few statements from the
lecture notes, for which only brief arguments were given.

(i) p. 153 − 154, Example 10.1: Prove the factorisation formula for the likelihood of
the observations $X_1, \ldots, X_n$ from an AR(1) process $X_t$:

$$p_{X_1, \ldots, X_n} = p_{X_1}(X_1) \prod_{t=2}^{n} p_Z(X_t - \phi X_{t-1}).$$

(ii) p. 155: Let $X_t$ and $Y_t$ define a hidden Markov model. Show that a sequence $(X_t, Y_t)$
is Markovian and that the joint density of $(X_1, Y_1, X_2, Y_2, \ldots, X_n, Y_n)$ is

$$p(x_1)p(x_2|x_1) \times \cdots \times p(x_n|x_{n-1}) \cdot p(y_1|x_1) \times \cdots \times p(y_n|x_n).$$

(iii) p. 155, Lemma 10.3: Give a formal proof of the first part of Lemma 10.3 (the proof
of the second part is already given in the proof of that lemma).