TWISTED FORMS OF LINEAR ALGEBRAIC GROUPS:
SOME ASPECTS OF COMPUTATION

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EINDHOVEN, NETHERLANDS

SERGEI HALLER

Abstract. This is a joint project with Arjeh M. Cohen, Scott H. Murray and
Don E. Taylor. We design algorithms for computation with groups of Lie type.
These are being implemented in the software package Magma.

Algorithms for computing with elements of untwisted groups of Lie type
are known and implemented in Magma. (Cohen, Murray, Taylor (2004); Haller
(2000); Riebeek (1998))

The twisted groups of Lie type are fixed point subgroups of untwisted
groups of Lie type. The possible twists for a given field extension and group
type are classified by Galois cohomology. We report on current work to make
Galois cohomology effective.

Let $G$ be a simple linear algebraic group defined over the field $k$. One step
in the process of computing the Galois cohomology of $G$ is to extend 1-cocycles
on a factor group $A/B$ to 1-cocycles on $A$, where $A$ is the group of algebraic
automorphisms of $G$ and $B$ the connected component of $A$. We will discuss
an algorithm to solve this problem, which is of interest in its own right.

Since the twisted groups are defined as fixed point subgroups, an intrinsic
definition is available. To be able to compute with the elements of these
groups, an extrinsic definition is needed. We will discuss an approach to find
root elements of the twisted groups of Lie type.